Turning the Interaction ON and OFF in Out of Equilibrium TFT

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In solid state

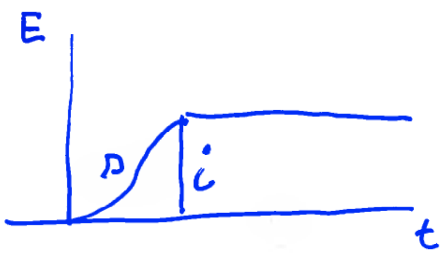
"on" and "off" very cleaar concepts

Example

electric field is created at $t=0$
with the help of man controlled device

instantaneously - much faster than relaxation processes

softly - comparable speed or slower than relaxation processes



$E(t) = \Theta(t-t_0)\, E$ instantan.

$= f(t-t_0)\, E$ softly

the same for turning off

I. D. Proceedings of Dubrovnik 2004.

Ref: I. D., Proceedings of XI Chris Engelbrecht Summer School in TF. Ed. J. Cleymans & H.B. Geyer, FG Schools edited by Springer 1998

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I. D., Phys. Rev. D 59 (1999), 125012-1-14

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I. D., Nucl. Phys. A702 (2002) 356C

I. D., hep-ph/0103025

D. Kuić, I. D. in preparation

Plan of the talk

Heavy ion collisions and out of eq
out of eq setup
derivation of the result
projected functions
convolutions
sign (p_0, ω_p)
⋮
numerics for "bare" fields
subtractions
numerics for "dressed" fields
conclusions

Left out of the main talk

general properties of Feynman diagrams
connection to Keldysh $(t_0 \to -\infty)$ time path
doubly projected functions [interaction turned "On" and "Off"]

resummations

Projected Functions

Two point function in finite time path formalism

$$A(x,y) = \Theta(x_0)\Theta(y_0)\bar{A}(x,y).$$

Retarded and advanced functions

$$A_R(x,y)) = \Theta(x_0 - y_0)\Theta(x_0)\Theta(y_0)\bar{A}(x,y),$$

$$A_A(x,y)) = \Theta(y_0 - x_0)\Theta(x_0)\Theta(y_0)\bar{A}(x,y).$$

Projected function

$$A^P(x,y)) = \Theta(x_0)\Theta(y_0)\bar{A}(x-y).$$

Projected retarded and advanced functions

$$A^P_{R(A)}(x,y)) = \Theta\left(\pm(x_0 - y_0)\right)\Theta(x_0)\Theta(y_0)\bar{A}(x-y),$$

Convolution product

$$[A * B](x,y) == \Theta(x_0)\Theta(y_0)\int_0^\infty dz_0 d^3z \bar{A}(x,z)\bar{B}(z,y).$$

Various convolution products projected functions:

$$[A_R * B_R](x,y) == \Theta(x_0 - y_0)\Theta(x_0)\Theta(y_0)\int_0^{x_0 - y_0} dz_0 d^3z \bar{A}(x-y-z)\bar{B}(z).$$

Convolution product of retarded projected functions is retarded projected function; the product of advanced projected functions is advanced projected function

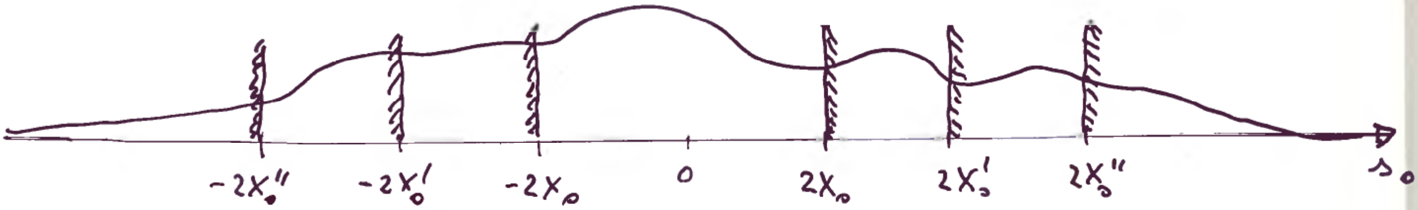
$$[A_A * B_A](x,y) = \Theta(y_0 - x_0)\Theta(x_0)\Theta(y_0)\int_0^{y_0 - x_0} dz_0 d^3z \bar{A}(-z)\bar{B}(z-y+x),$$

and the product of advanced projected function with retarded projected function is retarded projected function

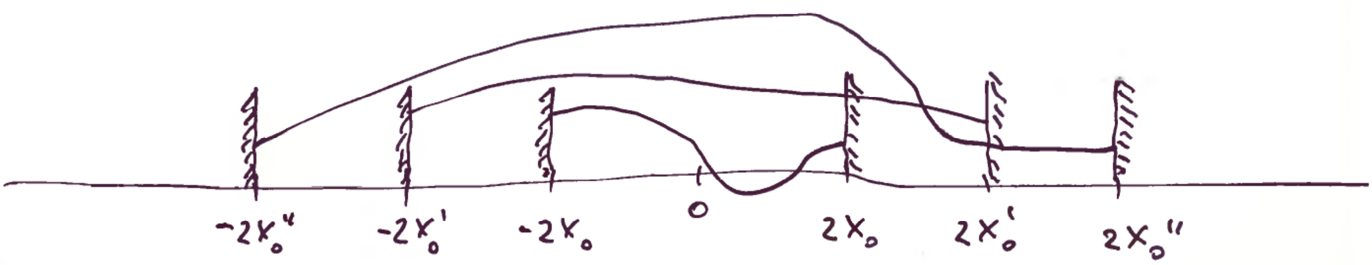
$$[A_A * B_R](x,y) = \Theta(x_0 - y_0)\Theta(x_0)\Theta(y_0)\int_0^\infty dz_0 d^3z \bar{A}(z)\bar{B}(x-y-z).$$

The product of retarded projected function with advanced projected function is not projected function

$$[A_R * B_A](x,y) = \Theta(x_0 - y_0)\Theta(x_0)\Theta(y_0)\int_0^{y_0} dz_0 d^3z \bar{A}(x-z)\bar{B}(z-y)$$



$A(X_0, s_0)$ projected function



$A(x_0, s_0)$ not projected function

Two-Point Functions

Switching-on the interaction at $t_i = 0$

Two-point function $G(x,y)$, transition to Wigner variables

(x,y),	four-vectors	$0 < x_0,\ y_0 < \infty$

$$\Updownarrow$$

(X,s),	$X = (x+y)/2,\ s = x-y,$	$0 < X_0,\ -2X_0 < s_0 < 2X_0$

$$G(x,y) = G(X + \frac{s}{2}, X - \frac{s}{2})$$

Fourier integral with respect to s_0, s_i and the inverse:

$$G(X + \frac{s}{2}, X - \frac{s}{2}) = (2\pi)^{-4} \int d^4p e^{-i(p_0 s_0 - \vec{p}\vec{s})} G(p_0, \vec{p}; X)$$

$$G(p_0, \vec{p}; X) = \int_{-2X_0}^{2X_0} ds_0 \int d^3s e^{i(p_0 s_0 - \vec{p}\vec{s})} G(X + \frac{s}{2}, X - \frac{s}{2})$$

Projection operator $P_{X_0}(s_0) = \Theta(2X_0 - s_0)\Theta(2X_0 + s_0)$

$$\int_{-2X_0}^{2X_0} ds_0 = \int_{-\infty}^{\infty} ds_0 P_{X_0}(s_0)$$

$$P_{X_0}(p_0, p_0') = \frac{1}{2\pi} \int_{-2X_0}^{2X_0} ds_0 e^{is_0(p_0 - p_0')} = \frac{1}{\pi} \frac{\sin(2X_0(p_0 - p_0'))}{p_0 - p_0'}$$

$$\lim_{X_0 \to \infty} P_{X_0}(p_0, p_0') = \delta(p_0 - p_0')$$

Homogeneity in space coordinates $\Leftrightarrow$ drop $\vec{X}$ dependence

Relation between the transforms at X_0 with finite and infinite carrier

$$G_{X_0}(p_0, \vec{p}) = \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0, p_0') G_\infty(p_0', \vec{p}; X_0)$$

Projected Functions

The projected function (PF) a very special two-point function:

Projected function does not depend on $\vec{X}$, it is a function of $(s_0, \vec{s})$ within the interval $-2X_0 < s_0 < 2X_0$ and identical to zero outside

$$F(x,y) = F(X + \frac{s}{2}, X - \frac{s}{2}) = \begin{pmatrix} F(s_0, \vec{s}) & -2X_0 < s_0 < 2X_0 \\ 0 & s_0 < -2X_0 \text{ or } 2X_0 < s_0 \end{pmatrix}$$

Fourier transforms of projected functions (FTPF's):

$$F_\infty(p_0, \vec{p}) = \int_{-\infty}^{\infty} ds_0 \int d^3 s e^{i(p_0 s_0 - \vec{p}\vec{s})} F(s_0, \vec{s})$$

$$F_{X_0}(p_0, \vec{p}) = \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0, p_0') F_\infty(p_0', \vec{p})$$

Note1: F_∞ does not depend on X_0!

Note2: F_{X_0} is determined by $F_\infty \Rightarrow F$ describes reversible process!

Examples of projected functions:

- poles in the energy plane
- R, A, and K components of free propagators
- R, A, and K components of one-loop self-energy
- results of some convolutions and resummations

Analytic properties of projected functions in the $X_0 \to \infty$ limit:

Define RETARDED (ADVANCED) functions:

$F_\infty(p_0)$ is retarded (advanced) function if it satisfies :
(1) the function of p_0 is analytic above (below) the real axis,
(2) the function goes to zero as $|p_o| \to \infty$ in the upper (lower) semiplane

desirable: PF R(A)-components $\Rightarrow$ R(A) functions

desirable: PF K-components $\Rightarrow$ sum of R and A functions

$$+\Theta(y_0 - x_0)\Theta(x_0)\Theta(y_0)\int_0^{x_0} dz_0 d^3z \bar{A}(x-z)\bar{B}(z-y).$$

The factors multiplying "Theta's" cannot be writen in a form which depends solely on $x - y$; thus the product $A_R * B_A$ contains two terms, of which one is retarded and the other advanced function, but they are not projected functions.

Propagator

Propagator in coordinate space ($0 < x_0$, $0 < y_0$):

Retarded component

$$G_R(x,y) = -G_{1,1} + G_{2,1} = \int d^4p \frac{-i}{p^2 - m^2 + 2i\epsilon p_0} e^{-ip(x-y)}$$

Keldysh component

$$G_K(x,y) = G_{1,1} + G_{2,2} = \int d^4p 2\pi\delta(p^2 - m^2)(1 + 2f(\omega_p))e^{-ip(x-y)}$$

G_R and G_K depend on $s = x - y$; vanish at $x_0 < 0$ or $y_0 < 0 \Rightarrow$ PF

Fourier transforms over infinite time interval:

$$G_{R,\infty}(p) = \frac{-i}{p^2 - m^2 + 2i\epsilon p_0}$$

$$G_{K,\infty}(p) = -(1 + 2f(\omega_p))\omega_p^{-1}\left(p_0 G_{R,\infty}(p) - p_0 G_{A,\infty}(p)\right)$$

Analyticity adoptions: (I. D. phys. Rev. D 59, 125012 (1999))

in	wrong	correct	alternative
G_R	$sgn(p_0)\epsilon$	$p_0\epsilon$	
G_K	$sgn(p_0)$	p_0/ω_p	ω_p/p_0

"ϵ" parameter regulations:

the limit $\epsilon \to 0$ should be taken last of all	
specially	$\lim_{X_0\to\infty} \exp(-X_0\epsilon) = 0$

Function $sign(p_0, \omega_p)$

An identity:

$$\delta(x-y) = \frac{i}{2\pi}\gamma(\frac{x}{y})[\frac{1}{x-y+i\epsilon} - \frac{1}{x-y-i\epsilon}] + \mathcal{O}(\epsilon^2),$$

where $\gamma(1) = 1$, analytic around $x/y = 1$. Generate next identity

$$\delta(p_0^2 - \omega_p^2) = \frac{i}{2\pi} sign(p_o, \omega_p)[\frac{1}{p_0^2 - \omega_p^2 + 2i\epsilon p_0} - \frac{1}{p_0^2 - \omega_p^2 - 2i\epsilon p_0}] + \mathcal{O}(\epsilon^2).$$

Instead of usual $sign(p_0)$ function, a new (user friendly) function $sign(p_0, \omega_p)$, which is an alternative between

$$sign(p_0, \omega_p) = sign(p_o), \frac{p_0}{\omega_p}, \frac{\omega_p}{p_0}, \left(\frac{p_0}{\omega_p}\right)^3, \left(\frac{\omega_p}{p_0}\right)^3, \ldots$$

$sign(p_0)$ usual, not recomended choice, nonanalytic at $p_0 = 0$
$\frac{p_0}{\omega_p}$ default choice, for convergent integrals
$\frac{\omega_p}{p_0}$ reduces the power of p_0 danger at $p_0 = 0$.
For all offered possibilities , the function $sign(p_0, \omega_p)$ at $p_0 = \pm\omega_p$ reduces to $sign(p_0)$ and the identity is valid. The choice of the appropriate form of $sign(p_0, \omega_p)$, should guarantee that in the perturbative expansion integrals over p_0 converge (in the way such, that two terms in Eq. (**??**) $G_{K,R}$ and $G_{K,A}$ could be treated separately) at $|p_0| = \infty$ and no additional singularities appear at finite p_0 (especially not at $|p_0| = 0$), This choice might be different for different terms. The difference between any two choices (when multiplied by $\delta(p_0^2 - \omega_p^2)$) is $\mathcal{O}(\epsilon^2)$. In the absence of pathology this difference vanishes in the $\epsilon \longrightarrow 0$ limit.
Having done proper choices one can integrate over p_0 as first. this results in manifestly retarded (advanced) functions.

Convolution Product

The convolution product of two two-point functions:

$$C = A * B \Leftrightarrow C(x,y) = \int dz A(x,z) B(z,y)$$

for A and B Fourier transforms of projected functions:

$$C_{X_0}(p_0,\vec{p}) = \int dp_{01} dp_{02} P_{X_0}(p_0, \frac{p_{01}+p_{02}}{2})$$
$$\frac{1}{2\pi}\frac{ie^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon} A_\infty(p_{01},\vec{p}) B_\infty(p_{02},\vec{p})$$

For A advanced or B retarded function product is FTPF!

$$C_{X_0}(p_0,\vec{p}) = \int dp_{01} P_{X_0}(p_0,p_{01}) A_\infty(p_{01},\vec{p}) B_\infty(p_{01},\vec{p})$$

Finite X_0 smearing of energy preserves uncertainty relations

$$\lim_{X_0\to\infty} C_{X_0}(p_0,\vec{p}) = A_\infty(p_0,\vec{p}) B_\infty(p_0,\vec{p})$$

general convolution product given by gradient expansion:
(note we assumed the homogeneity in space coordinates)

$$C_{X_0}(p_0,\vec{p}) = e^{-i\diamondsuit} A_{X_0}(p_0,\vec{p}) B_{X_0}(p_0,\vec{p}), \quad \diamondsuit = \frac{1}{2}(\partial^A_{X_0}\partial^B_{p_0} - \partial^A_{p_0}\partial^B_{X_0})$$

Equal-Time Two-Point Functions

reduction of two-point functions to equal time two-point functions

$$x_0 = y_0 = t \qquad \Leftrightarrow \qquad X_0 = t,\ s_0 = 0$$

obtained by inverse Wigner transform as

$$G(t,0,\vec{p}) = \frac{1}{2\pi}\int dp_0 G_{X_0=t}(p_0,\vec{p})$$

An example - average of number operator

$$< 2N_{\vec{p}}(t) + 1 >= \omega_p G_K(t,0,\vec{p}) = \frac{\omega_p}{2\pi}\int dp_0 G_{t,K}(p_0,\vec{p})$$

The case of bare fields

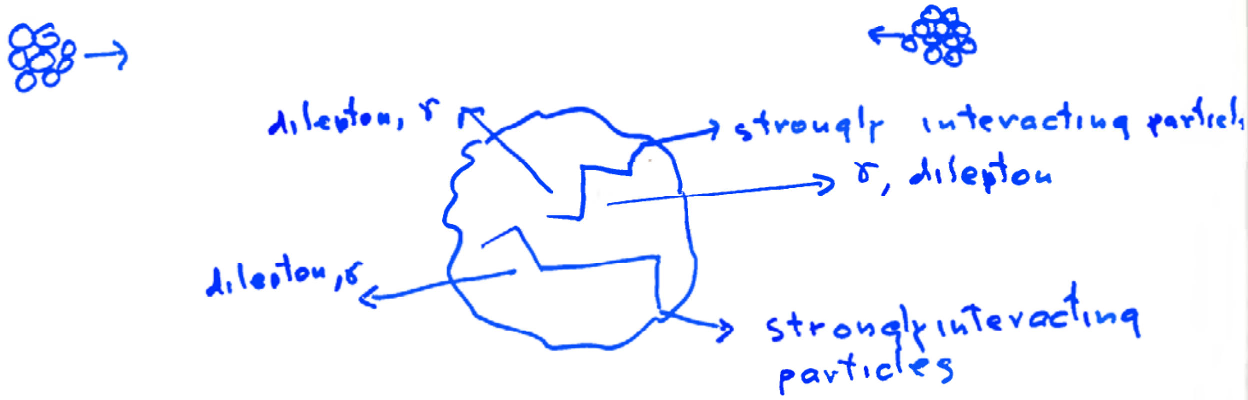
$$< 2N^0_{\vec{p}} + 1 >= \frac{\omega_p}{2\pi}\int dp_0 G^0_{X_0,K}(p) = 1 + 2f(\omega_p)$$

Right hand side time independent
Equal-time function from retarded WTPF ($s_0 = 0 \Leftrightarrow \lim_{s_0\to+0}$)

$$G_R(t,0,\vec{p}) = \frac{1}{2\pi}\int dp_0 G_{X_0,R}(p_0,\vec{p})$$
$$= \frac{1}{2\pi}\int dp_0 \int dp_{01} P_{X_0}(p_0,p_{01}) G_{\infty,R}(p_{01},\vec{p})$$
$$= \frac{1}{2\pi}\int dp_{01} G_{\infty,R}(p_{01},\vec{p}) = const(\vec{p}).$$

The integral over WTPF $G_{X_0,R}$ does not change with time
Important but expected result: projected function determined by $X_0 = +\infty$; it cannot describe irreversible processes.
non-WTPFs $\rightarrow$ byproduct of pinching.

<u>heavy ion collision at ultrahigh energy</u>



<u>model:</u>

at the time t_0 (set $t_0 = 0$!) one can consider that QCD plasma has formed!

characteristics: $\boxed{t = 0}$

quark distribution $f_q(\vec{p}) \equiv f$

antiquark " $f_{\bar{q}}(\vec{p}) \equiv \bar{f}$ initial ρ density matrix

gluon " $f_g(\vec{p})$

no. photons $f_\gamma(\vec{p}) \equiv 0$

time evolution through QFT!

<u>measure photons</u> at the time t

$$N(t) = \sum_p a_p^+(t) \; a_p(t) \qquad \text{only transverse}$$

$$= \lim_{t' \to t} a_p^+(t') \; a_p(t)$$

$$\mathrm{Tr}\, N(t)\, \rho = \lim_{t' \to t} G_K(t, t')$$

<u>contributions</u> of the order α^1

Schwinger - Dyson equations

$$\mathcal{G}_R = G_R + i\, G_R * \Sigma_R * \mathcal{G}_R$$

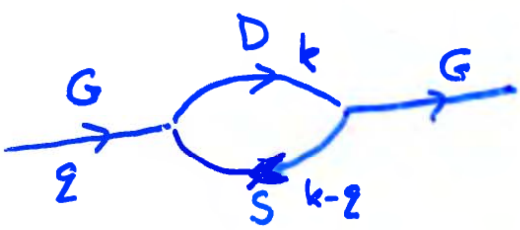
$$\mathcal{G}_A = \mathcal{G}_A + i\, G_A * \Sigma_A * \mathcal{G}_A$$

$$\mathcal{G}_K = G_K + i\, G_R * \Sigma_K * \mathcal{G}_A + i\, G_K * \Sigma_A * \mathcal{G}_A + i\, G_R * \Sigma_R * \mathcal{G}_K$$

$*$ convolution product

$$\Sigma_R = -i\,\frac{g^2}{2}\int\frac{d^4k}{(2\pi)^4}\Big\{D_R(k)\left[S_R(k-q) - S_K(k-q)\right] + \left[D_A(k) - D_K(k)\right]S_A(k-q)\Big\}$$

$$\Sigma_K = i\,\frac{g^2}{2}\int\frac{d^4k}{(2\pi)^4}\Big\{\left[D_R(k)+D_A(k)\right]\left[S_R(k-q)+S_A(k-q)\right] + D_K(q)\,S_K(k-q)\Big\}$$



Elimination of Pinching in the Single Self-Energy Insertion Approximation

The Keldysh component in the single self-energy insertion approximation

$$G_K = G_{Kp,R} + G_{Kp,A} + G_{Kr}$$

$$G_{Kp,R} = -iG_R * \bar{\Omega}_R * G_R, \quad G_{Kp,A} = iG_A * \bar{\Omega}_A * G_A$$

$$G_{Kr} = h(G_R - G_A) + iG_R * h\Sigma_R * G_R$$

$$-iG_A * h\Sigma_A * G_A$$

where we have introduced short notation: $\bar{\Omega}_{R(A)} = h\Sigma_{R(A)} + \Omega_{R(A)}$. $G_{Kp,R}$ and $G_{Kp,A}$ pinchlike contributions; G_{Kr} free from pinching. In full details Ω_R term (the dependence on $\vec{p}$ not shown):

$$G_{Kp,R} = -iG_R * \bar{\Omega}_R * G_A$$

$$G_{X_0,Kp,R}(p_o, \vec{p}) = -i \int dp_{01} dp_{02} dp_{03} P_{X_0}(p_0, \frac{p_{01}+p_{03}}{2}) G_R(p_{01})$$

$$\frac{i}{2\pi} \frac{e^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon} \bar{\Omega}_{\infty,R}(p_{02}) \frac{i}{2\pi} \frac{e^{-iX_0(p_{02}-p_{03}+i\epsilon)}}{p_{02}-p_{03}+i\epsilon} G_A(p_{03})$$

Integrate over p_{02} by closing the integration path from above. The only singularity closed is situated at $p_{01} + i\epsilon$ (note the care for ϵ's):

$$G_{X_0,Kp,R}(p_0, \vec{p}) = -i \int dp_{01} dp_{03} P_{X_0}(p_0, \frac{p_{01}+p_{03}}{2}) G_R(p_{01})$$

$$\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon) \frac{i}{2\pi} \frac{e^{-iX_0(p_{01}-p_{03}+2i\epsilon)}}{p_{01}-p_{03}+2i\epsilon} G_A(p_{03})$$

Integrate over p_{03} by closing the integration path from above. The singularities closed are situated at $p_{01}+2i\epsilon$ and at $\pm\omega_p+i\epsilon$.

$$G_{X_0,Kp,R}(p_0,\vec{p})$$

$$= -i\int dp_{01}P_{X_0}(p_0,p_{01})G_R(p_{01})\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon)G_A(p_{01}+2i\epsilon)$$

$$-\frac{1}{2\omega_p}\int dp_{01}G_R(p_{01})\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon)$$

$$\times\sum_{\lambda=-1}^{1}\lambda P_{X_0}(p_0,\frac{p_{01}+\lambda\omega_p}{2})\frac{e^{-iX_0(p_{01}-\lambda\omega_p+i\epsilon)}}{p_{01}-\lambda\omega_p+i\epsilon}$$

From the definitions of G_R and G_A one observes that $G_A(p_{01}+2i\epsilon)=G_R(p_{01})$ so that all the functions appearing in above expression are retarded. There is no pinching in integration over $p_{0,1}$. To p_0 can be safely given finite positive imaginary part and the $p_{0,1}$ integral will be well defined. There is no pinching in the final expression.
But we have got functions directly depending on time X_0 i.e., the non-FTPF functions, which one cannot convolute further in the elegant way we were using here.
Finally

$$G_{X_0,K}(p_0,\vec{p})$$

$$= 2Im\Big(\int dp_{01}P_{X_0}(p_0,p_{01})G_R(p_{01})\Omega_{\infty,R}(p_{01}+i\epsilon)G_R(p_{01})$$

$$-i\frac{1}{2\omega_p}\int dp_{01}G_R(p_{01})\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon)$$

$$\times\sum_{\lambda=-1}^{1}\lambda P_{X_0}(p_0,\frac{p_{01}+\lambda\omega_p}{2})\frac{e^{-iX_0(p_{01}-\lambda\omega_p-i\epsilon)}}{p_{01}-\lambda\omega_p+i\epsilon}\Big)$$

it gives just the constant distribution function. In higher order of perturbation expansion one expects that also the distribution function changes with time. The above result tells us that it happens only if there is non-FTPF contribution!
Now we come back to the result of cancellation of pinching, we add G_{Kr} to $G_{Kp,R}$ and $G_{Kp,A}$ and obtain

$$G_{X_0,K}(p_0,\vec{p}) = 2Im\Big(\int dp_{01} P_{X_0}(p_0,p_{01}) G_R(p_{01}) \Omega_{\infty,R}(p_{01}+i\epsilon) G_R(p_{01})$$

$$-i\frac{1}{2\omega_p}\int dp_{01} G_R(p_{01})\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon)$$

$$\sum_{\lambda=-1}^{1} \lambda P_{X_0}(p_0,\frac{p_{01}+\lambda\omega_p}{2})\frac{e^{-iX_0(p_{01}-\lambda\omega_p-i\epsilon)}}{p_{01}-\lambda\omega_p+i\epsilon}\Big).$$

This expression is second order contribution to the generalized distribution function. As we shall see by integration over p_0,

$$\int dp_0 G_{X_0,K}(p_0,\vec{p}) = 2Im\Big(\int dp_{01} G_R(p_{01})\bar{\Omega}_{\infty,R}(p_{01}+i\epsilon) G_R(p_{01})$$

$$e^{-iX_0(p_{01}+i\epsilon)}(\cos X_0\omega_p + i\frac{p_{0,1}}{\omega_p}\sin X_0\omega_p)\Big).$$

thanks to the presence of non-FTpf the result depends on X_0.
The fact that FTPF do not contribute to Eq. (4.20), throws new light on our approach: pinchlike contributions (i.e. those containing convolution products of both retarded and advanced components) are necessary to obtain nontrivial time dependence. As this fact will reappear in the other expressions (even calculation of retarded and advanced compononts from two-loop or more complicated Feynman diagrams we may conclude that indeed, the pinchlike expressions represent "the body of evidence" that very important information is left "ill-defined" in the formulation using Keldysh time path.

$$= \frac{2}{\pi(2\pi)^3}\int_{-\infty}^{\infty} dp_0 R(p_0)\frac{1-\cos(p_0-p)t}{(p_0-p)^2}. \tag{14}$$

The connection is established by

$R(p_0) = -Im\tilde{\Sigma}_{\infty,<}(p_0)$ As explained we prefer to use Keldysh component $Im\tilde{\Sigma}_{\infty,K}(p_0)/2 = -Im\tilde{\Sigma}_{\infty,K,R}(p_0)$. They are related by $\Sigma_< = \frac{-\Sigma_K-\Sigma_R+\Sigma_A}{2}$

3.1 Preliminaries

Note here that the presence of the term proportional to 1 in Eq. (13) is optional. Indeed for this term one can close the integration contour from above and find that it vanishes. But it simplifies the calculation. Even more, as $\Sigma_{\infty,K,R} \propto |p_0|^0$ as $|p_0| \to \infty$ in the case of $q\bar{q} \to \gamma$, we can add "null" terms up to the power p_0^2. The choice $1 - .5sin^2pt(p^2-p_0^2)/p^2$ improves the behaviour of integrand near the singular points $p_0 = \pm p$.

Owing to the factor e^{-itp_0} in (13) the integration over p_0 cannot be performed by closing the integration path from above. Instead we have to deform the path and close it from below.

In doing so we obtain cut contribution coming solely from $Im\tilde{\Sigma}$ which is peaked near $p_0 = \pm p$. The factor [ ] vanishes at $p_0 = \pm p$, thus the contribution comes from the nearby region.

$$[1-e^{-itp_0}(\cos tp + i\frac{p_0}{p}\sin tp)] = 1 - \cos p_0 t \cos pt$$

$$-\frac{p_0}{p}\sin p_0 t \sin pt - i(\frac{p_0}{p}\cos p_0 t \sin pt - \sin p_0 t \cos pt)$$

$$= 1 - \cos(p_0-p)t + \frac{p-p_0}{p}\sin p_0 t \sin pt$$

$$-i[\sin(p-p_0)t - \frac{p-p_0}{p}\cos p_0 t \sin pt]. \tag{15}$$

Only real part contributes; as it is symmetric, only symmetric part of $Im\tilde{\Sigma}_R$ (where we understand that by "symmetry" operation one remains on the same side of the real axis) contributes (factor of 2 comes from the disconti-

3 Main Expressions

In the standard approach one starts with Eq. (7), applies to it either G_R^{-1} from the left, or G_A^{-1} from the right, then performs two gradient expansions (to the order $O((\partial)^2)$) for two convolution products, adds and subtracts the results, and obtains kinetic equations. These are solved by Dynamical Renormalisation Group methods.

Our approach is more direct: as we can perform two convolution products in an exact manner, we consider Eq. (7) as the solution of kinetic equations and simplify it further.

We start with Eq. (6)

$$
\begin{aligned}
< N_{\vec{p}}(t) >= & \frac{d\mathcal{N}(t)}{d^3pd^3x}(2\pi)^3 \\
= & \frac{\omega_p}{4\pi}\int dp_o[G_{t,K}(p_o,\vec{p}) - G_{0,K}(p_0,\vec{p})]. \qquad (11)
\end{aligned}
$$

Now we consult Eq. (4.11) from (ID3)

$$
\begin{aligned}
& \int dp_0 G_{t,K}(p_0,\vec{p}) = -2Im[\int dp_0 G_{\infty,R}(p_0,\vec{p}) \\
& \tilde{\Sigma}_{\infty,K,R}(p_0+i\epsilon,\vec{p})G_{\infty,R}(p_0,\vec{p})e^{-it(p_0+i\epsilon)} \\
& (\cos t\omega_p + i\frac{p_0}{\omega_p}\sin t\omega_p)]. \qquad (12)
\end{aligned}
$$

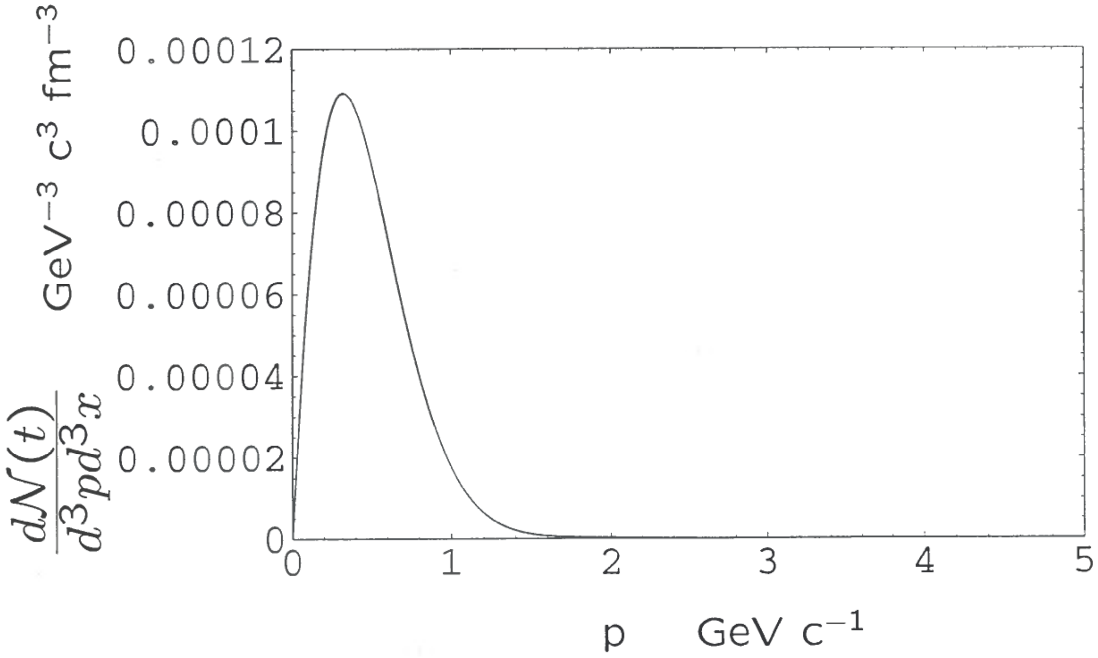
Thus

$$
\begin{aligned}
\frac{p < N_{\vec{p}}(t) >}{(2\pi)^3} = p\frac{d\mathcal{N}(t)}{d^3pd^3x} = & \frac{2}{\pi(2\pi)^3}\frac{p^2}{2}\int_{-\infty}^{\infty} dp_0 Im\Big(-\tilde{\Sigma}_{\infty,K,R}(p_0,\vec{p}) \\
& \frac{1}{(p_0^2-p^2+2i\epsilon p_0)^2}[1-e^{-itp_0}(\cos tp + i\frac{p_0}{p}\sin tp)]\Big). \qquad (13)
\end{aligned}
$$

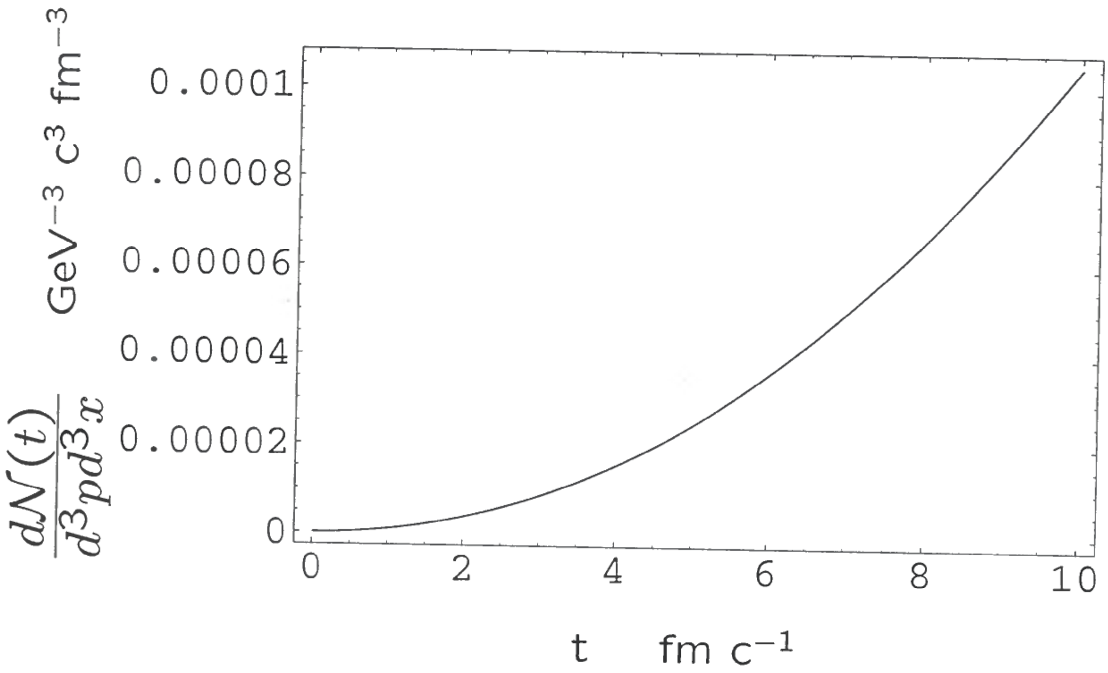
Where we have decomposed Σ_K to retarded and advanced part $\Sigma_K = -\Sigma_{K,R} + \Sigma_{K,A}$. We have also introduced $\tilde{\Sigma}_K = -\tilde{\Sigma}_{K,R} + \tilde{\Sigma}_{K,A}$, where $\tilde{\Sigma}_{K,R(A)} = \Sigma_{K,R(A)} - sign(p_0,p)\Sigma_{R(A)}$. We set here $\omega_p = |\vec{p}| = p$.

To understand the WB paper we give their Eq. (10):

$$
\frac{p < N_{\vec{p}} >}{(2\pi)^3} = p\frac{d\mathcal{N}(t)}{d^3pd^3x}
$$



Photon number density as a function of momentum for $t = 10$ fm/c. Parametar $T = 0.3$ GeV. Quark masses (u and d) are equal zero.



Photon number density as a function of time for $p =$ 0.4 GeV/c. Parametar $T = 0.3$ GeV. Quark masses (u and d) are equal zero.