Renormalized 2PI resummation

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Outlines

- Perturbation theory at finite temperature and in nonequilibrium
- IR divergences and resummations
- Problems with renormalizability
- Resummation as a renormalization scheme
- Momentum dependent resummations; 2PI resummation as a scheme
- 6D Φ^3 theory with renormalized 2PI resummation
- Conclusions

Perturbation theory at finite temperature and in nonequilibrium

1.) (Non)equilibrium quantum fields in Path Integral

For simplicity we will work with bosonic fields.

We should compute expectation values of correlation functions local operators, in Heisenberg picture

$$\langle A(t_1,\mathbf{x}_1)\dots A(t_n,\mathbf{x}_n)\rangle_\rho = \mathrm{Tr}\,\rho\, A(t_1,\mathbf{x}_1)\dots A(t_n,\mathbf{x}_n)$$

To represent the trace: equal time commutation relation of field operators

$$[\Phi(t,\mathbf{x}),\Pi(t,\mathbf{x}')] = i\delta^{(3)}(\mathbf{x}-\mathbf{x}'), \qquad [\Phi(t,\mathbf{x}),\Phi(t,\mathbf{x}')] = 0, \qquad (\hbar = 1)$$

$\hat{\Phi}$ commute $\quad\Rightarrow\quad \exists\,|\Phi,t\rangle$ eigenfunction-system: $\hat{\Phi}(t,\mathbf{x})\,|\Phi,t\rangle = \Phi(\mathbf{x})\,|\Phi,t\rangle\,.$

QM analogy: $\hat{q} \to \hat{\Phi}, \quad |q\rangle \to |\Phi\rangle$

$\Rightarrow$ the representation of time evolution is analog, too

$$\left\langle \Phi' \left| e^{-iH(t-t_i)} \right| \Phi \right\rangle = \int_{\Phi}^{\Phi'} \mathcal{D}\Phi \, e^{iS[\Phi]}.$$

With operator insertions in time ordered way $t > t_1 > \ldots > t_n > t_i$

$$\left\langle \Phi' \left| e^{-iH(t-t_1)} A_1 \, e^{-iH(t_1-t_2)} \ldots A_n \, e^{-iH(t_n-t_i)} \right| \Phi \right\rangle$$
$$= \int_{\Phi}^{\Phi'} \mathcal{D}\Phi \, A_1(t_1) \ldots A_n(t_n) \, e^{iS[\Phi]}.$$

Correlation function of operators:

Heisenberg picture $A(t) = e^{iH(t-t_i)} A e^{-iH(t-t_i)}$
for time ordered product: $t_f > t_1 > \ldots > t_n > t_i$

$$\langle \Phi' \left| A_1(t_1) \ldots A_n(t_n) \right| \Phi \rangle =$$

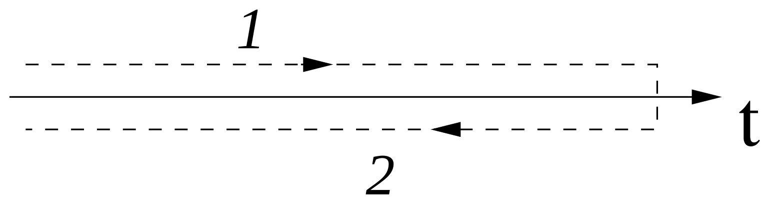
$$\left\langle \Phi' \left| e^{iH(t_1-t_i)} A_1 \, e^{-iH(t_1-t_2)} A_2 \, e^{-iH(t_2-t_3)} \ldots A_n \, e^{-iH(t_n-t_i)} \right| \Phi \right\rangle =$$

$$\sum_{\Phi_f} \left\langle \Phi' \left| e^{iH(t_f-t_i)} \right| \Phi_f \right\rangle \left\langle \Phi_f \left| e^{-iH(t_f-t_1)} A_1 \, e^{-iH(t_1-t_2)} \ldots A_n e^{-iH(t_n-t_i)} \right| \Phi \right\rangle$$

Second term: completely time ordered $\Rightarrow$ can be represented by path integral
First term: completely *anti-time oredered*, time flows in "reversed direction"
because of $\int_{t_f}^{t_i} dt \, L = -S$

$$\left\langle \Phi' \left| e^{iH(t_f-t_i)} \right| \Phi_f \right\rangle = \int_{\Phi_f}^{\Phi'} \mathcal{D}\Phi \, e^{-iS[\Phi]} \Big|_{\text{reversed time ordering}} .$$

The two terms together define a common path integral contour



Keldysh (or CTP≡ closed time path) contour time oredering:

$$\langle \Phi' \,| \mathrm{T} A_1(t_1) \ldots A_n(t_n) |\, \Phi \rangle = \int_{\Phi}^{\Phi'} \mathcal{D}\Phi_1 \mathcal{D}\Phi_2 \, e^{iS[\Phi_1] - iS[\Phi_2]} \, A_1^{(1)}(t_1) \ldots A_n^{(1)}(t_n).$$

- The operators live in the first section. If any of them lives on the second section:

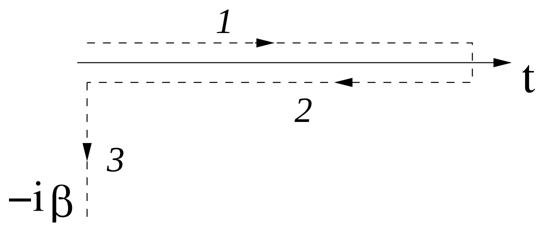
$$\int_{\Phi}^{\Phi'} \mathcal{D}\Phi_1 \mathcal{D}\Phi_2 \, e^{iS[\Phi_1] - iS[\Phi_2]} \, B^{(2)} \, A^{(1)} = \left\langle \Phi' \left| (\mathrm{T}^* \hat{B})(\mathrm{T} \hat{A}) \right| \Phi \right\rangle$$

- The complete expectation value

$$\left\langle \mathrm{T} \hat{A} \right\rangle_{\rho} = \mathrm{Tr}\, \rho \, \mathrm{T} \hat{A} = \sum_{\Phi, \Phi'} \langle \Phi | \rho | \Phi' \rangle \left\langle \Phi' \left| (\mathrm{T} \hat{A}) \right| \Phi \right\rangle$$

How to take into account ρ?

- in equilibrium: $e^{-\beta H} = e^{-i(-i\beta H)} \quad \Rightarrow \quad 0 \to -i\beta$ imaginary time evolution. In Keldysh formalism: 3. section added to the contour (Matsubara contour).



- we can build in the path integral; it affects only the configurations at $t = t_i$.

$$\left\langle \mathrm{T}\hat{A} \right\rangle_\rho = \int \mathcal{D}\Phi_1 \mathcal{D}\Phi_2 \, e^{iS[\Phi_1] - iS[\Phi_2] + iV[\Phi_1 - \Phi_2]} \, A[\Phi_1]$$

where

$$iV(\Phi) = \sum_{n=1}^{\infty} \frac{1}{n!} d^4x_1 \dots d^4x_n \Phi(x_1) \dots \Phi(x_n) \, \mathrm{Tr} \, \varrho \mathcal{F}(x_1) \dots \mathcal{F}(x_n) \Big|_{\mathrm{conn}}$$

$$\mathcal{F}(t, \mathbf{k}) = \delta(t - t_i) \, \Pi(t_i, \mathbf{k}) + i\partial_t \delta(t - t_i) \, \Phi(t_i, \mathbf{k}).$$

Equilibrium $t_i \to -\infty$, initial correlations die out, only the quadratic (gaussian) part which counts $\quad \Rightarrow \quad$ only the free propagators get modified!

Propagators:

initial conditions connect Φ_1 and Φ_2 $\quad\Rightarrow\quad$ propagator is a matrix

$$\begin{aligned}
iG^{11}(x,y) &= \left\langle \mathrm{T}_c \Phi^{(1)}(x)\Phi^{(1)}(y)\right\rangle = \left\langle \mathrm{T}\Phi(x)\Phi(y)\right\rangle_\rho \\
iG^{22}(x,y) &= \left\langle \mathrm{T}_c \Phi^{(2)}(x)\Phi^{(2)}(y)\right\rangle = \left\langle \mathrm{T}^*\Phi(x)\Phi(y)\right\rangle_\rho \\
iG^{12}(x,y) &= \left\langle \mathrm{T}_c \Phi^{(1)}(x)\Phi^{(2)}(y)\right\rangle = \left\langle \Phi(y)\Phi(x)\right\rangle_\rho \\
iG^{21}(x,y) &= \left\langle \mathrm{T}_c \Phi^{(2)}(x)\Phi^{(1)}(y)\right\rangle = \left\langle \Phi(x)\Phi(y)\right\rangle_\rho .
\end{aligned}$$

Redundant: $G^{11} + G^{22} = G^{12} + G^{21}$ $\quad\Rightarrow\quad$ it is worth to introduce different fields

$$\Phi_1 = \Phi_r + \frac{\Phi_a}{2}, \qquad \Phi_2 = \Phi_r - \frac{\Phi_a}{2} \qquad .$$

Here $iG^{aa} = 0$ automatically, and

$$\begin{aligned}
iG^{ra} &= iG^{11} - iG^{12} = \Theta(t-t')\left\langle [\Phi(x),\Phi(x')]\right\rangle_\rho \\
iG^{ar} &= iG^{11} - iG^{21} = -\Theta(t'-t)\left\langle [\Phi(x),\Phi(x')]\right\rangle_\rho \\
iG^{rr} &= \tfrac{1}{2}(iG^{12} + iG^{21})
\end{aligned}$$

G^{ra} is the retarded, G^{ar} is the advanced propagator
G^{rr} is called Keldysh propagator.

In free bosonic theory, in equilibrium

$$G^{ra}(k) = \frac{1}{k^2 - m^2 + i\mathrm{sgn}(k_0)\varepsilon}, \quad iG^{rr}(k) = \left(\frac{1}{2} + n(E_k)\right) \, 2\pi\delta(k^2 - m^2)$$

where $n(E)$ is the Bose-Einstein distribution.

In equilibrium it is true in general (KMS relation)

$$G^{21}(k) = e^{\beta k_0} G^{12}(k) \quad \Rightarrow \quad iG^{rr}(k) = \left(\frac{1}{2} + n(k_0)\right) \varrho(k),$$

where $\varrho = -2\,\mathrm{Im}\,G^{ra}$.

$$\mathrm{Tr}\, e^{-\beta H} A(t) B(0) = \mathrm{Tr}\, e^{-\beta H} A(t) e^{\beta H} e^{-\beta H} B(0) = \mathrm{Tr}\, e^{-\beta H} B(0) A(t + i\beta)$$
$$\Rightarrow \quad G^{21}_{AB}(t) = G^{12}_{AB}(t + i\beta) \quad \Rightarrow \quad G^{21}_{AB}(k_0) = e^{\beta k_0} G^{12}_{AB}(k_0).$$

2.) Perturbation theory

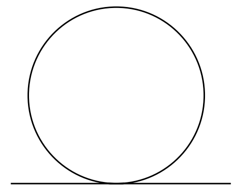
idea: split the system as free theory & interactions
but: what is the "free theory"? what are the particles?
$\Rightarrow$ this depends on the environment (eg. temperature, time)
$\Rightarrow$ the best we can hope is to derive a gap or an evolution equation.

$$\Downarrow$$

even the determination of the leading order is complicated!

If despite we try to build up the perturbation theory on a bad free theory
$\Rightarrow$ we encounter infrared divergences!

Simplest example: in Φ^4 theory, in equilibrium the one-loop mass corrections



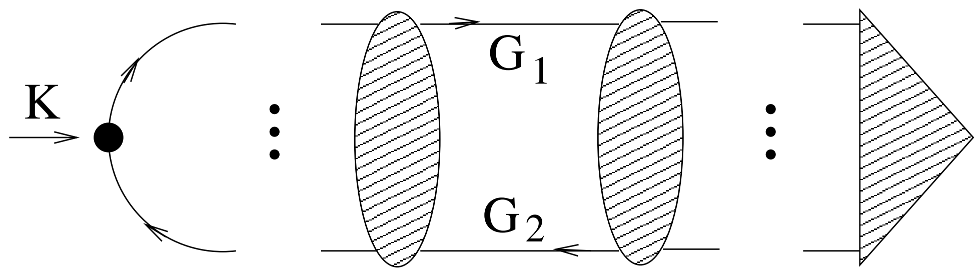
$$\Rightarrow \quad \frac{\Delta m^2}{m^2} \sim \frac{\lambda T^2}{m^2}.$$

- At high temperatures the correction can be larger than the tree level mass
- At higher loops corrections of order $\left(\frac{\lambda T^2}{m^2}\right)^n$ come

$\Rightarrow$ perturbation theory is unreliable!

IR divergences in non-equilibrium case

Example of a higher order diagram for Φ^2 correlation function (ladder diagram)



(shaded ellipses can be self-energy corrections or 4-point functions; triangle means initial conditions.)

Product of a retarded and an advanced propagator: poles are on the lower and upper complex half plane, respectively $\Rightarrow$ if multiplied with the same momentum, we obtain double poles

$$G_R(Q-\frac{K}{2})G_A(Q+\frac{K}{2}) = \frac{i\varrho(Q-\frac{K}{2})}{2QK} + \mathrm{nonsing}$$

divergent for $K \to 0$ $\Rightarrow$ pinch singularity

At higher orders all rungs introduce a new pinch singular piece
$\Rightarrow$ in N-th order $\sim g^{2N} k_0^{-N}$ (taking $\mathbf{k} = 0$).
In real space

$$\int_{-\infty}^{\infty} \frac{dk_0}{2\pi} g^{2N} k_0^{-N} e^{-ik_0 t} \sim g^{2N} t^{N-1}$$

secular terms $\Rightarrow$ perturbation theory fails for long times.

$\Downarrow$

Direct perturbation theory does not work, at least in the free theory we should take into account the effects of the environment
$\Rightarrow$ we should trace the vacuum state and the particle propagator

2PI approximation

Tracing the vacuum

We represent the vacuum as a background field: $\phi = \bar{\Phi} + \varphi$, where $\bar{\Phi} = \langle \Phi(x) \rangle$ $\Rightarrow$ fluctuation vev is zero $\langle \varphi \rangle = 0$.

Evaluating the path integral with given background $\Rightarrow$ effective action.

Determination of the background field equation of motion (EoM) from the extremum of the effective action

$$0 = \frac{\delta \Gamma}{\delta \bar{\Phi}} = \left\langle \frac{\partial \mathcal{L}}{\partial \varphi} \right\rangle$$

In Φ^4 theory: $$0 = \partial^2 \Phi + m^2 \Phi + \frac{\lambda}{6} \Phi^3 + \frac{\lambda}{2} \Phi \left\langle \varphi^2 \right\rangle + \frac{\lambda}{6} \left\langle \varphi^3 \right\rangle .$$

- near equilibrium we can compute the expectation values
- generic structure in perturbation theory:

$$0 = \frac{\delta S_{\rm cl}}{\delta \bar{\Phi}} + J_{\rm ind}[G],$$

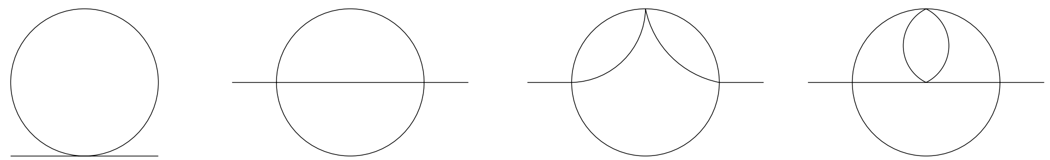
where the induced current is functional of the propagators.

Tracing the free excitations

Schwinger-Dyson equations

$$(\partial^2 + m^2)G + \Sigma * G = 0,$$

where Σ is the self-energy (matrix in case of the Keldysh formalism). In the self-energy we assume that we use exact propagators (2PI approximation)
$\Rightarrow$ in perturbative expansion only skeleton diagrams should be taken into account. Some examples:



Local approximation: $\Sigma(k) = \Delta m^2$ $\Rightarrow$ in case of finite temperature this defines a thermal mass.

Self-consistent equations: $G[\Sigma] \quad \Leftrightarrow \quad \Sigma[G]$

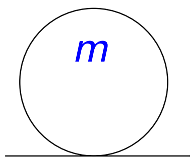
$$\Downarrow$$

Numerical solution of 2PI equations.

Entangled UV and IR divergences

Example: propagator with temperature dependent mass.

Self-energy correction comes from the tadpole diagram; in dim. reg.:



$$= T_B(m^2) = \frac{m^2}{16\pi^2}\left[-\frac{1}{\varepsilon} + \gamma_E - 1 + \ln\frac{m^2}{4\pi\mu^2}\right] + \frac{1}{2\pi^2}\int_m^\infty d\omega\,\sqrt{\omega^2 - m^2}\,n(\omega).$$

- in perturbation theory needs mass counterterm: $\delta m^2 = \frac{m^2}{16\pi^2}\frac{1}{\varepsilon}$.
- at finite temperature 2PI approximation the mass depends on the temperature $m^2 \to m^2(T) \quad \Rightarrow \quad$ divergence $-\frac{m^2(T)}{16\pi^2}\frac{1}{\varepsilon}$

$$\Downarrow$$

either we have a T-dependent counterterm, or an unbalanced UV divergency.

Why renormalization and resummation are in conflict?

Renormalization:

- we need it to cure the UV divergences of the theory
- we group contributions according to their UV relevance
- perturbation theory with counterterms consistent method: the counterterms are taken into account at higher orders to cancel the overall divergences of the lower order diagrams $\Rightarrow$ reorganization of pert. series from the point of view of the *bare theory*
  – subdivergences are canceled systematically (BPHZ, forest formula)
  – finite parts of the counterterms: renormalization scheme

Resummation:

- we need it if we encounter sensitivity at finite (zero) energy scale
  1PI (Schwinger-Dyson equations), daisy, HTL, leading log, etc.
- we group contributions according to their IR relevance
  $\Rightarrow$ usually different from the UV relevance; "normal" and counterterm diagrams are not necessarily of the same relevance.
- resummation reshuffles perturbation series $\Rightarrow$ separates "normal" and counterterm diagrams $\Rightarrow$ UV divergences do not cancel at a given order

$$\Downarrow$$

For UV consistent resummation we have to resum counterterm diagrams, too.

A different view on resummation

Resummation $\Rightarrow$ singled out reference PT ("unresummed" PT)

In general: PT should be fitted to the environment

Example: O(N) model SSB. Two PTs, one with excitations with masses m^2, the other with excitations $m_G^2 = 0$, $m_H^2 = -2m^2$.
Neither is "better": applicable in different physical situation.

$\Rightarrow$ instead "resummation" use the most adequate PT

- Fundamental theories are IR finite (IR observables independent of UV cutoff).
- Perturbation theory is IR finite order by order

$\Rightarrow$ only renormalized PTs should be applied!

- Not fully fixed: we can use different finite parts $\equiv$ different schemes

$\Rightarrow$ finite parts should be fitted to environment

Renormalization scheme as resummation

1.) Zero temperature

Different finite parts $\Rightarrow$ different schemes
Zero temperature example: $\overline{\mathrm{MS}}$ scheme and mass-shell scheme (S)

Φ^4 model, one loop complete self energy

$$\Sigma_{\mathrm{S}}(m) = m^2, \qquad \Sigma_{\overline{\mathrm{MS}}}(m) = m^2 + \frac{\lambda m^2}{32\pi^2} \ln \frac{m^2}{\mu^2}$$

To describe the same physics the two schemes must be related by RG transformation.
Condition: bare Lagrangian is the same!

$$Z^2 m_{\mathrm{bare}}^2 = m_{\overline{\mathrm{MS}}}^2 + \delta m_{\overline{\mathrm{MS}}}^2 = m_S^2 + \delta m_S^2$$

where

$$\delta m_{\overline{\mathrm{MS}}}^2 = -\frac{\lambda m_{\overline{\mathrm{MS}}}^2}{32\pi^2} \left[-\frac{1}{\varepsilon} + \gamma_E - 1 - \ln 4\pi \right] \quad \delta m_S^2 = -\frac{\lambda m_S^2}{32\pi^2} \left[-\frac{1}{\varepsilon} + \gamma_E - 1 + \ln \frac{m_S^2}{4\pi\mu^2} \right]$$

Up to $\mathcal{O}(\lambda)$ order divergences cancel, and

$$m^2_{\overline{\mathrm{MS}}} = m^2_S - \frac{\lambda m^2_S}{32\pi^2} \ln \frac{m^2_S}{\mu^2}.$$

For any n-point function

$$O^{(n)}_{\overline{\mathrm{MS}}}(m_{\overline{\mathrm{MS}}}) = O^{(n)}_S(m_S(m_{\overline{\mathrm{MS}}}))(1 + \mathcal{O}(\lambda))$$

$\Rightarrow$ contains higher loop contributions!

$$\Downarrow$$

Changing scheme is equivalent to resum higher order diagrams!

2.) Finite temperature

same example at finite temperature: $\overline{\mathrm{MS}}$ scheme and mass-shell scheme (S)
Φ^4 model one loop complete self energy

$$\Sigma_{\mathrm{S}}(m) = m^2, \qquad \Sigma_{\overline{\mathrm{MS}}}(m) = m^2 + \frac{\lambda m^2}{32\pi^2} \ln \frac{m^2}{\mu^2} + \frac{\lambda}{4\pi^2} J(m),$$

where $J(m) = \int_m^\infty d\omega \sqrt{\omega^2 - m^2}\, n(\omega)$

- $\overline{\mathrm{MS}}$ scheme IR sensitive, since 1-loop correction $\sim \lambda T^2 \gg m^2$ at high T.
- on-mass-shell (S) scheme temperature dependent, but safe

$$\Downarrow$$

Use the IR safe mass-shell scheme, but translate results to $\overline{\mathrm{MS}}$ with RG transformation!

The same condition as before now reads

$$m^2_{\overline{\mathrm{MS}}} = m_S^2 - \frac{\lambda m_S^2}{32\pi^2} \ln \frac{m_S^2}{\mu^2} - \frac{\lambda}{4\pi^2} J(m_S)$$

a gap equation, can be solved for m_S. At high T the most relevant part

$$m_S^2 = m^2_{\overline{\mathrm{MS}}} + \frac{\lambda T^2}{24} + \ldots$$

For any n-point function

$$O^{(n)}_{\overline{\mathrm{MS}}}(m_{\overline{\mathrm{MS}}}) = O^{(n)}_S(m_S(m_{\overline{\mathrm{MS}}}))(1 + \mathcal{O}(\lambda))$$

$\Rightarrow$ containes higher loop contributions, ie. resummation effect, but without entangling UV and IR degrees of freedom

$$\Downarrow$$

finite thermal mass resummation.

Momentum dependent resummations

Generic 2PI resummation: propagator cannot be represented by a simple mass term

can a renormalization scheme accomplish this resummation?

Generalize mass-shell scheme: regularized self energy at one loop level:

$$\Sigma(k,m) = m^2 + \bar{\Sigma}(k,m) + \delta m^2.$$

Choose $\delta m^2 = -\bar{\Sigma}(k,m) \quad \forall k$!

- No self energy correction: $\Sigma(k,m) = m^2$
  $G^{-1}(k) = k^2 - m^2 = G_0^{-1}(k) \quad \Rightarrow$ free propagator is exact!

  $\Rightarrow$ 2PI scheme

- 2PI scheme is temperature and momentum dependent $\Rightarrow$ we should translate the results to $\overline{\mathrm{MS}}$ via RG transformation
  Condition: bare Lagrangian is the same

$$m^2 - \bar{\Sigma}(k,m) = m^2_{\overline{\rm MS}} + \delta m^2_{\overline{\rm MS}}$$

right hand side momentum independent $\Rightarrow$ m^2 should be momentum dependent to compensate the mom. dependence of $\bar{\Sigma}$:

$$m^2(k) - \bar{\Sigma}(k,m(k)) = m^2_{\overline{\rm MS}} + \delta m^2_{\overline{\rm MS}}$$

$\Rightarrow$ 2PI gap equation for $m^2(k)$.

BUT: not finite in this form! Terms like $m^2(k) \sim m^2 \ln k,\ k^2 \ln k$ are generated

$\Rightarrow$ $\bar{\Sigma}(k,m)$ contains divergences not occuring in $\delta m^2_{\overline{\rm MS}}$.

Regularization

No new divergences, if for asymptotic momenta $m^2(k) \sim m_R^2 + \mathcal{O}(k^{-\gamma})$ because: in Φ^4 theory the most singular diagram is the tadpole.

$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2(p)} = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_R^2 - \mathcal{O}\left(p^{-\gamma}\right)} \longrightarrow$$

$$\longrightarrow \int \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2 - m_R^2} + \mathcal{O}\left(\frac{p^{-\gamma}}{(p^2 - m_R^2)^2} \right) \right].$$

The last term is finite if $\gamma > 0$.

Split $\bar{\Sigma} = \bar{\Sigma}^{\mathrm{div}} + \bar{\Sigma}^{\mathrm{sing}} + \bar{\Sigma}^{\mathrm{reg}}$: divergent, singular (growing with k) and regular pieces.

- Divergence structure should be scheme-independent, $\bar{\Sigma}^{\mathrm{div}} := -\delta m^2_{\overline{\mathrm{MS}}}$.
- $\bar{\Sigma}^{\mathrm{sing}}(k, m) \sim k^2 \ln k/\mu,\ m^2 \ln k/\mu \quad \Rightarrow \quad$ identifiable

Choosing $\delta m^2(k) = \delta m^2_{\overline{\mathrm{MS}}} - \bar{\Sigma}^{reg}(k,m)$

$\Rightarrow$ $m^2(k)$ is regular, ie. goes to constant for asympt. momenta.

$\Rightarrow$ we do not generate new divergences, gap equation is finite

$$m^2(k) = m^2_{\overline{\mathrm{MS}}} + \bar{\Sigma}^{\mathrm{reg}}(k,m).$$

In this case the exact propagator is not exactly the free propagator

$$G^{-1}(k) = k^2 - m^2(k) - \Sigma^{\mathrm{sing}}(k,m)$$

but perturbation theory is free from IR divergences.

6D Φ^3 theory 2PI resummation at finite temperature

Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial\Phi)^2 - \frac{m^2}{2}\Phi^2 - \frac{g}{6}\Phi^3 + \text{counterterms}$$

2PI resummation $\Rightarrow$ $m^2(k)$.

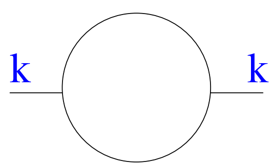
We use real time formalism in R/A basis $\Rightarrow$ generic equilibrium mass matrix can be characterized by $m_R(k)$ retarded mass.

Gap equation to solve:

$$m_R^2(k) = m_{\overline{\text{MS}}}^2 + \Sigma_R^{\text{reg}}(k)$$

where

$$\Sigma_R^{\text{reg}}(k) =$$



$$\Big|_{\text{regular}}$$

Logic of the solution of the gap equation

1. $\varrho_0(k) \quad \Rightarrow \quad \mathrm{Im}\,\Sigma_R(k) = -\frac{g^2}{2}\int \frac{d^6p}{(2\pi)^6}\left(\frac{1}{2}+n(p_0)\right)\varrho_0(p)\,\varrho_0(k-p)$

2. $\mathrm{Im}\,\Sigma_R(k) \quad \Rightarrow \quad \mathrm{Im}\,\Sigma_R^{\mathrm{reg}}(k) = \mathrm{Im}\,\Sigma_R(k) + \frac{g^2}{2(4\pi)^3}\Theta(k^2-M^2)\left(\frac{k^2}{6}-m^2\right)$

3. $\mathrm{Im}\,\Sigma_R^{\mathrm{reg}}(k) \quad \Rightarrow \quad \Sigma_R^{\mathrm{reg}}(k) = \int\limits_{-\infty}^{\infty} \frac{d\omega}{\pi}\frac{-\mathrm{Im}\,\Sigma_R^{\mathrm{reg}}(\omega,\mathbf{k})}{k_0-\omega+i\varepsilon}$

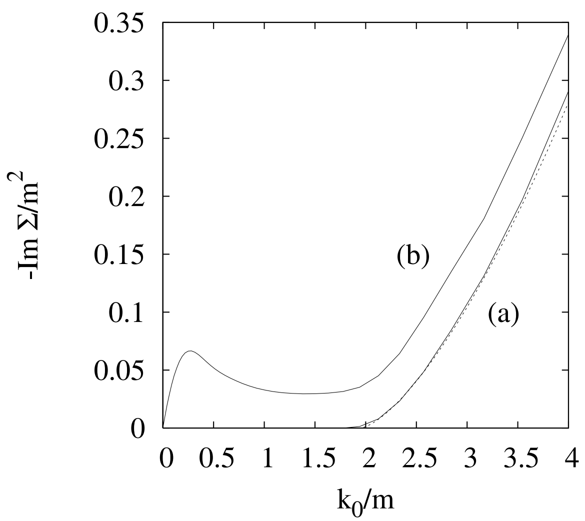
4. $\Sigma_R^{\mathrm{reg}}(k) \quad \Rightarrow \quad m_R^2(k) = m_{\overline{\mathrm{MS}}}^2 + \Sigma_R^{\mathrm{reg}}(k)$

5. $m_R^2(k) \quad \Rightarrow \quad \varrho_0(k) = \frac{-2\,\mathrm{Im}\,m_R^2(k)}{(k^2-\ \mathrm{Re}\,m_R^2(k))^2+(\,\mathrm{Im}\,m_R^2(k))^2}$

6. go to step one until the process converges. . .

Results

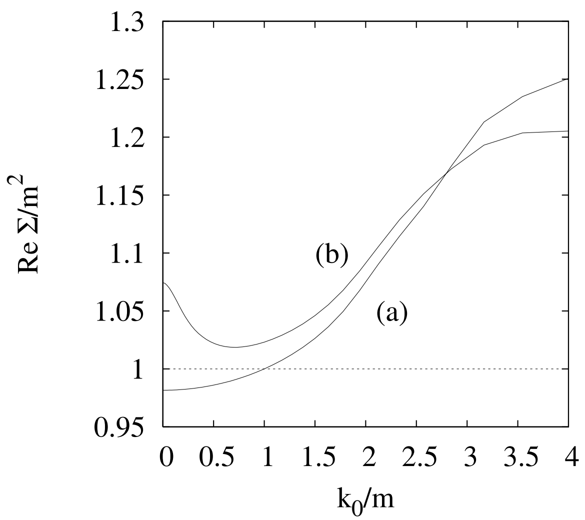
self-energy imag. part



- (a): $T = 0$, (b): finite T
- $T = 0$: zero below 2-particle threshold, fits well to perturbative curve (below 3-particle threshold)
- finite T: nonzero everywhere; smeared out Landau damping for $k_0 < k$; between the Landau damping region and zero temperature cut nearly constant.

parameters: $g = 20,\ T = m$ and $0,\ k = 0.25m.$

self-energy real part

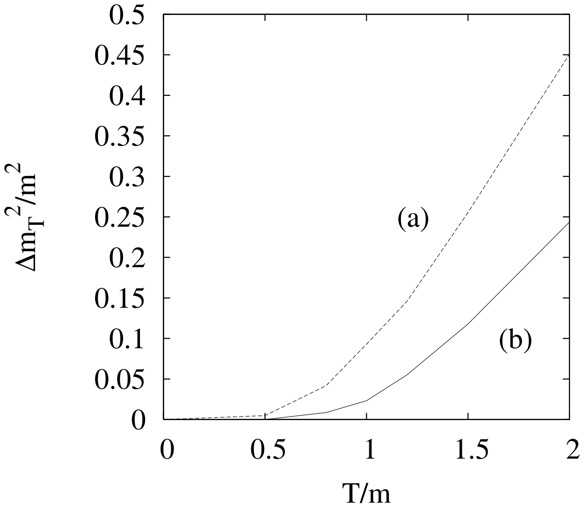


- (a): $T = 0$, (b): finite T
- $T = 0$: renormalization condition: on-shell mass is m $\Rightarrow$ $\Sigma_{\text{tot}}(k^2 = m^2) = m^2$.
- finite T: larger than zero temperature result at $k_0 < 2.5m$ $\Rightarrow$ positive "thermal mass" correction.
- Big difference between the thermal mass defined at $k_0 = 0$ (Debye mass) and $k_0 = m$ (quasiparticle mass)!

parameters: $g = 20$, $T = m$ and 0, $k = 0$.

Quasiparticle properties

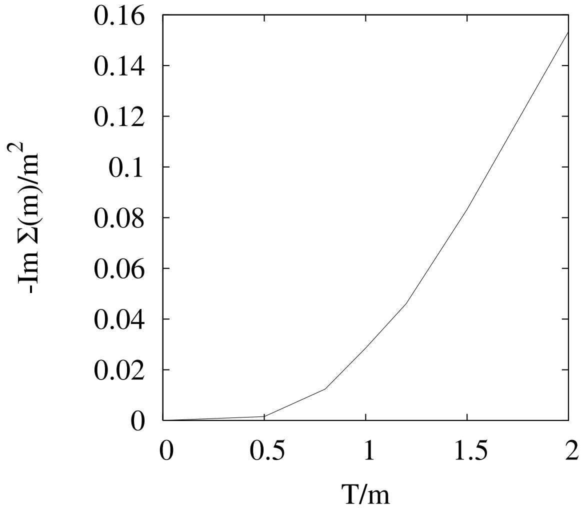
thermal mass correction



- (a): $k_0 = 0$, Debye- or screening mass, (b) $k_0 = m$, quasiparticle mass.
- Big difference $\Rightarrow$ a single effective mass term cannot be a satisfactory description.
- For $T/m > \mathcal{O}(1)$ both curves $\sim aT^2 + b$, in the small temperature regime the curve is shallower

parameters: $g = 20,\ k = 0.$

thermal mass correction



parameters: $g = 20,\ k = 0.$

- Imaginary part of the self energy on the mass shell: resummation effect.
- For $T/m > \mathcal{O}(1)$ quadratic $\sim aT^2 + b$, for small T it is shallower
  $\Rightarrow$ quasiparticle damping $\sim T$ at high temperatures.

Conclusions

- pure perturbation theory cannot be applied at finite temperature or in non-equilibrium $\Rightarrow$ resummation is needed
- adapting the free theory to the environment $\Rightarrow$ 2PI resummation. Gap equation or evolution equation for the vacuum (background field) and the free excitation properties.
- Problems with renormalizability: apparently temperature dependent or momentum dependent divergences. Reason: resummation separates the "normal" and counterterm diagrams. Solution: resum counterterm diagrams together with the IR sensitive normal diagrams.
- Renormalized perturbation theory automatically ensures consistency, finite parts of the counterterms (scheme) is free to choose $\Rightarrow$ use schemes to accomplish resummation.
  Finite temperature on-mass-shell scheme $\equiv$ thermal mass resummation.
- Momentum dependence: choose momentum dependent finite parts. For consistency mass should go to constant at asymptotic momenta. 2PI resummation $\equiv$ 2PI scheme; only regular part can be resummed.

- 6D Φ^3 theory, 2PI resummation: Debye mass and quasiparticle mass is rather different $\Rightarrow$ single mass approximation is not appropriate. On-shell damping: beyond pure 1-loop perturbation theory.