Nonclassical interaction-free detection of objects in a monolithic total-internal-reflection resonator

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We show that with an efficiency exceeding 99% one can use a monolithic total-internal-reflection resonator to ascertain the presence of an object without transferring a quantum of energy to it. We also propose an experiment on the probabilistic meaning of the electric field that contains only a few photons. © 1997 Optical Society of America [S0740-3224(97)01106-5]

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1. INTRODUCTION

Quantum-optical measurements can exhibit the outstanding and completely nonclassical feature of detecting an object without transferring a single quantum of energy to it. For example, after the second beam splitter of a properly adjusted Mach–Zehnder interferometer one can always put a detector in such a position that it almost never detects a photon. If it does, then we are certain that an object has blocked one path of the interferometer. A measurement that makes use of this quantum-mechanical feature was designed by Elitzur and Vaidman.[1,2] For dramatic effect they assumed that the object is a bomb and showed that it would explode in only 50% of the tests if an asymmetrical beam splitter were employed. The tests, of course, always have to be carried out with single photons. Kwiat *et al.*[3,4] proposed a setup that is based on "weak repeated tests" carried out by single photons. They aim at reducing the probability of exploding the bomb to as close to 0% as possible and propose using two identical cavities weakly coupled by a highly reflective beam splitter. Because of the interference the probability for a photon inserted into the first cavity to be located in it approaches 0, while the probability that it is found in the second cavity approaches 1, at a certain time T_N. However, if there is an opaque object (a bomb) in the second cavity the probabilities are reversed. So if we insert a detector into the first cavity we almost never get a click if there is no absorber in the second cavity and almost always get a click if there is an absorber there. The probability of exploding the bomb when there is a detector in the first cavity approaches 0. The drawback of the proposal is that it is apparently difficult to carry it out. In particular, inserting a detector into a cavity at a given time and introducing a single photon into a cavity are by no means simple tasks.

In Section 2 we propose a simple, efficient, and easily feasible interaction-free experiment—with an arbitrarily high probability of detecting the bomb without exploding it—that is based on the resonance in a single monolithic total-internal-reflection resonator that recently showed a significant practical advantage and high efficiencies. The proposal assumes a pulsed or a continuous-wave (cw) laser beam and a properly cut isotropic crystal. The additional advantage of the present proposal over the proposals of Kwiat *et al.*[3,5] is that the latter infer information on the presence or absence of the bomb in the system from the presence or absence of a detector click, whereas we get the information from the firing of appropriate detectors.

First we perform a classical calculation. As is well known, there is a formal correspondence between the classical and the quantum descriptions in the sense that classical quantities, e.g., the amount of energy absorbed by a bomb, are identical to quantum-mechanical ensemble averages. When the absorbed energy is, on average, only a small fraction of an energy quantum $h\nu$, this means in reality that in most cases no absorption takes place at all and that in only a few cases one photon is actually absorbed. This is a consequence of the corpuscular nature of light. In other words, interpreting classical intensities as quantum-mechanical probabilities of finding a photon allows us to translate our classical results into quantum-mechanical predictions.

Our elaboration shows that, in accordance with quantum theory, an overwhelming number of photons detected by a detector indicating a presence of a bomb in the system will not exchange any energy with the bomb. Only in rare cases will a photon transfer an energy quantum $h\nu$ to the bomb. Nevertheless, on average, there is no difference between the quantum and the classical pictures. This means that many other parallels between the two pictures, such as wave amplitude calculations, coherence time and length, optical resonator calculations, and cavity photon decay time, should be preserved. On the other hand, in one-photon quantum interference the amplitudes of electric field have a probabilistic meaning only, and one might wonder whether classical reasoning would give a correct answer. For example, Weinfurter *et al.*[6] and Fearn *et al.*[7] proposed an experiment in which one would decide whether sudden changing of boundary conditions of frustrated downconverted photons affects photons instantaneously or after a (classically untenable) delay that would allow all parts of the system (atoms emitting downconverted photons) to receive the information on the changed conditions.

In Section 3 we propose an experiment in which one would decide whether sudden changing of boundary conditions (object–no object) imposed on photon paths redirects the photons (into detector D_r instead of into detector D_t and vice versa) instantaneously (classically untenable) or after a delay that would allow for sufficiently many round trips to build up the interference.

2. INTERACTION-FREE EXPERIMENT

The setup of the experiment is shown in Fig. 1. The experiment uses an uncoated monolithic total-internal-reflection resonator (MOTIRR) coupled to two triangular prisms by frustrated total internal reflection (FTIR).[8,9] A squared MOTIRR requires a relative refractive index with respect to the surrounding medium of $n > 1.41$ to confine a beam to the resonator (the angle of incidence is 45°). If, however, another medium (in our case the right-hand triangular prism in Fig. 1) is brought within a distance of the order of the wavelength, the total reflection within the resonator will be frustrated, and a fraction of the beam will tunnel out from the resonator. Depending on the dimension of the gap and the polarization of the incident beam, one can well define reflectivity R within the range from 10^{-5} to 0.99995.[9,10] The main advantage of such a coupling—in comparison with coated resonators—is that the losses are extremely small, as little as 0.3%. In the same way a beam can tunnel into the resonator through the left-hand triangular prism in Fig. 1, provided that the condition $n > 1.41$ is fulfilled for the prism too. The incident laser beam is chosen to be polarized perpendicularly to the incident plane to give a unique reflectivity for each photon. The faces of the resonator are polished spherically to give a large focusing factor. A round-trip path for the beam is created in the resonator, as shown in Fig. 1. A cavity is cut in the resonator and filled with an index-matching fluid to reduce losses. (Before we carry out the measurement we have to wait until the fluid comes to a standstill to avoid possible destabilization of the phase during round trips of the beam.) Now, if there is an object in the cavity in the round-trip path of the beam in the resonator, the incident beam will be almost totally reflected (into D_r), and if there is no object the beam will be almost totally transmitted (into D_t).

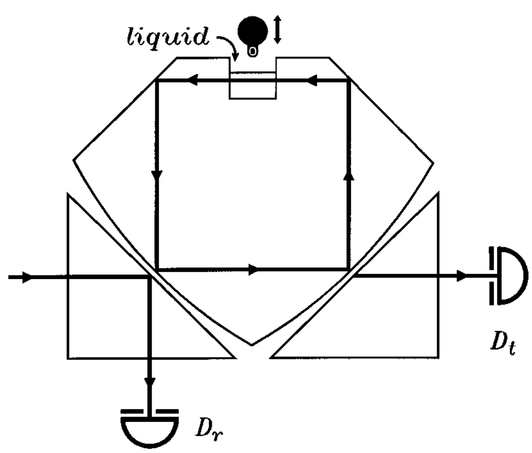

Fig. 1. Setup of the proposed experiment. For the free round-trips shown within the total-internal-reflection resonator the incident laser beam tunnels in and out to yield the zero intensity of the reflected beam; i.e., detector D_r does not react even when the incoming FTIR approaches 1. However, when the bomb is immersed in the (index-matching) liquid practically the whole incoming beam is reflected into D_r.

We start with a formal presentation of the experiment. Our aim is to determine the intensity of the beam reaching detector D_r first when the crystal is at resonance and then when an opaque object (bomb) is in the round-trip path in the crystal. A portion of a polarized incoming beam of amplitude $A(\omega)$ is totally reflected from the inner surface of the left-hand coupling prism (see Fig. 1) and suffers a phase shift that ranges from 0° (for the critical angle $\theta_1 = \arcsin n$) to 180° (for $\theta_1 = 0°$), where θ_1 is the incident angle at the prism inner surface.[11] When we couple the ring cavity (MOTIRR) to the prism, a part of the incoming beam will tunnel into the MOTIRR, the reflected beam will suffer a new phase shift δ, and the complex-field-reflection coefficient will read as

$$re^{i\delta} = \left[1 - \frac{2 \sin \delta_1 \sin \delta_2}{\cosh 2bx - \cos(\delta_1 + \delta_2)}\right] \times \exp\left(\frac{\sin \delta_1 \sinh 2bx}{\cos \delta_1 \cosh 2bx - \cos \delta_2}\right), \quad (1)$$

where δ_1 and δ_2 are the phase shifts of the waves at coupled surfaces of the prism and the MOTIRR, respectively, x is the gap between the prism and the MOTIRR, and $b = (2\pi/\lambda_0)(n_1{}^2 \sin^2 \theta_1 - n_2{}^2)^{1/2}$, where n_1 and n_2 are the refractive indices of the prism and the MOTIRR, respectively, and λ_0 is the vacuum wavelength. Thus detector D_r rotated at an angle δ with respect to the incoming plane will receive the incoming power of the incident beam attenuated by $|r|^2$.

At resonance the waves leaving the cavity after some round trips in the cavity will add up to destructive interference to make the reflected power vanish. Formally, this process can be described as follows.[9] The ratio of the reflected and the incoming powers is

$$\eta = 1 - \frac{c(x)}{1 + \left[\frac{2\mathscr{F}}{\pi} \sin \frac{\delta(x) + \phi}{2}\right]^2}, \quad (2)$$

where ϕ is the total phase shift acquired by the wave during one round trip, $\mathscr{F}$ is the finesse, and $c(x)$ is the coupling, given by

$$c(x) = \frac{(1 - e^{2\alpha})[1 - |r(x)|^2]}{[1 - e^{-\alpha}|r(x)|]^2}, \tag{3}$$

where α is a constant describing the round-trip losses. The reflected power will vanish when the MOTIRR is impedance matched, i.e., when the gap is adjusted to satisfy $|r(x_m)| = e^{-\alpha}$. Then $c(x_m) = 1$ and $\phi = 2N\pi - \delta$, where N is an integer, so $\eta = 0$.

To understand this result we sum up the contributions that originate from round trips in the resonator to the reflected wave. The portion of the incoming beam of amplitude $A(\omega)$ reflected into plane determined by δ is described by the amplitude $B_0(\omega) = -A(\omega)\sqrt{R}$, where $R = |r|^2$ is reflectivity. The transmitted part will travel around the resonator guided by one FTIR (at the face next to the right-hand prism) and by two proper total internal reflections. After a full round trip the following portion of this beam joins the directly reflected portion of the beam by tunneling into the left-hand prism: $B_1(\omega) = A(\omega)\sqrt{1-R}\sqrt{R}\sqrt{1-R}e^{i\psi}$. $B_2(\omega)$ contains three FTIR's and so on; each subsequent round trip contributes to a geometric progression that gives the reflected amplitude

$$B_n(\omega) = A(\omega)\sqrt{R}\{-1 + (1-R)e^{i\psi}[1 + Re^{i\psi} + (Re^{i\psi})^2 + \ldots]\} = \sum_{i=0}^{n} B_i(\omega), \tag{4}$$

where $\psi = (\omega - \omega_{\mathrm{res}})T$ is the phase added by each round trip. Here ω is the frequency of the incoming beam, T is the round-trip time, and ω_{res} is the selection frequency corresponding to a wavelength that satisfies $\lambda = L/k$, where L is the round-trip length of the cavity and k is an integer. On summing up the round-trip contributions we have taken into account that (because of the above condition imposed on the total phase shift ϕ) all the contributions must lie in the reflected-wave plane and that their amplitudes must carry the sign opposite that of the reflected wave, $-A(\omega)\sqrt{R}$ to cancel at resonance, $\psi = 0$.

To get a quick insight into the physics of the experiment let us first look at plane waves $[A(\omega) = A_0]$. The limit of $B_n(\omega)$ yields the total amplitude of the reflected beam:

$$B_r(\omega) = \lim_{n\to\infty} B_n(\omega) = -A_0\sqrt{R}\,\frac{1 - e^{i\psi}}{1 - Re^{i\psi}}. \tag{5}$$

We see that, for any $R < 1$ and $\omega = \omega_{\mathrm{res}}$, i.e., if nothing obstructs the round trip of the beam, we get no reflection at all [i.e., no response from D_r (see Fig. 1)]. When a bomb blocks the round trip and R is close to 1, then we get almost total reflection. In terms of single photons (which we can obtain by attenuating the intensity of a laser until the chance of having more than one photon at a time becomes negligible) the probability that detector D_r will react when there is no bomb in the system is 0. A response from D_r means interaction-free detection of a bomb in the system. The probability of the response is R, the probability of making a bomb explode by our device is $R(1-R)$, and the probability of a photon's exiting into detector D_t is $(1-R)^2$.

We achieve a more realistic experimental approach by looking at two possible sources of individual photons: a cw laser and a pulsed laser. For a pulsed laser we make use of a Gaussian wave packet $A(\omega) = A\exp[-\tau^2(\omega - \omega_{\mathrm{res}})^2/2]$, where τ is the coherence time, which obviously must be significantly longer than the round-trip time T. For a cw laser we use $A(\omega) = A\delta(\omega - \omega_{\mathrm{res}})$ (i.e., we assume a well-stabilized laser beam locked at ω_{res}) with a negligibly small linewidth. The incident wave is described by

$$E_i^{(+)}(z, t) = \int_0^\infty A(\omega)\exp[i(kz - \omega t)]\mathrm{d}\omega \tag{6}$$

and the reflected wave by

$$E_r^{(+)}(z', t) = \int_0^\infty B(\omega)\exp[i(kz' - \omega t)]\mathrm{d}\omega. \tag{7}$$

The energy of the incoming beam is the energy flow integrated over time:

$$I_i = \int_{-\infty}^\infty E_i^{(+)}(z, t)E_i^{(-)}(z, t)\mathrm{d}t = \int_0^\infty A(\omega)A^*(\omega)\mathrm{d}\omega. \tag{8}$$

The energy of the reflected beam is given analogously by $I_{r;n} = \int_0^\infty B_n(\omega)B_n{}^*(\omega)\mathrm{d}\omega$. Thus for the both types of laser the ratio of energies η as a function of the number of round trips n is given by

$$\eta_n = \frac{I_i}{I_{r;n}} = R\left\{1 - \frac{1-R}{1+R}\left[R^{2n} - 1 + 2\sum_{j=1}^{n}(1 + R^{2n-2j+1})R^{j-1}\Phi(j)\right]\right\}, \tag{9}$$

where $\Phi(j) = 1$ for cw lasers and $\Phi(j) = \exp(-j^2a^{-2}4^{-1})$ for pulsed lasers, where $a \equiv \tau/T$. This expression is obtained by mathematical induction from the geometric progression of the intensities of the amplitudes given by Eq. (4) and a subsequent integration over wave packets. It follows from Eq. (5) that the series with $\Phi(j) = 1$ converges, so the series with $\Phi(j) = \exp(-j^2a^{-2}4^{-1})$ converges as well. For the latter $\Phi(j)$ a straightforward calculation yields

$$\lim_{n\to\infty} \eta_n = \frac{\int_0^\infty B_r(\omega)B_r{}^*(\omega)\mathrm{d}\omega}{\int_0^\infty A(\omega)A^*(\omega)\mathrm{d}\omega} = 1 - (1-R)^2 \times \frac{\int_0^\infty \dfrac{\exp[-\tau^2(\omega - \omega_{\mathrm{res}})^2]\mathrm{d}\omega}{1 - 2R\cos[(\omega - \omega_{\mathrm{res}})\tau/a] + R^2}}{\int_0^\infty \exp[-\tau^2(\omega - \omega_{\mathrm{res}})^2]\mathrm{d}\omega}, \tag{10}$$

where $B_r(\omega)$ is from Eq. (5). In Fig. 2 the three upper curves represent three sums—obtained for three different values of a—that converge to values (shown as filled

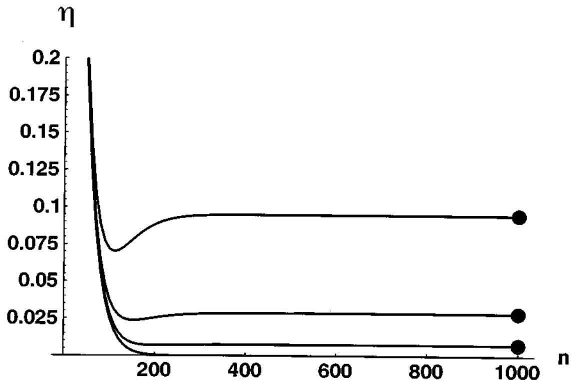

Fig. 2. Realistic values of η [ratio of the incoming and reflected powers, given by Eq. (9)] for $R = 0.98$. For pulsed lasers the three upper curves represent sums from Eq. (9) [with $\Phi(j) = \exp(-j^2 a^{-2} 4^{-1})$] as a function of n for $a = 100$, $a = 200$, and $a = 400$ (top to bottom), where $a \equiv \tau/T$ is a ratio of the coherence time τ and the round-trip time T; filled circles represent the corresponding values of η obtained from Eq. (10). For cw lasers the lowest curve represents the sum given by Eq. (9) [with $\Phi(j) = 1$] as a function of the number of round trips n.

circles) obtained from Eq. (10). The figure shows that a and n are closely related in the sense that the coherence length should always be long enough ($a > 200$) to permit sufficiently many (at least 200) round trips.

A cw laser oscillating in a single transverse mode has the advantage of excellent frequency stability (to 10 kHz, and with some effort even to 1 kHz, in the visible range) and therefore a very long coherence length (as much as 300 km).[12] This yields the zero intensity at detector D_r as with the plane waves described above. The only disadvantage of using a cw laser is that we have to modify the setup by adding a gate that determines a time window (1 ms$-$1 μs $<$ coherence time) within which the input beam arrives at the crystal and that allows the intensity in the cavity to build up. Then the intensity of the beam should be lowered to make it probable that only one photon will appear within the time window. We start each testing by opening the gate, and when either D_r or D_t fires, or when the bomb explodes, the testing is over. Of course, detectors might fail to react, but this is not an essential problem because the single-photon detector efficiency has already reached 85%, which would result in a bigger time window but a low probability of activating the bomb. In any case, the possibility of a 300-km coherence length does not leave any doubt that a real experiment can be carried out successfully. It should be emphasized that we get information on the presence or the absence of the bomb in any case from a detector click; hence we need no additional information that a photon has actually arrived at the entrance surface. This is a great advantage over the above-mentioned proposal by Kwiat *et al.*[3,5] in which the absence of the bomb is, in fact, inferred from the absence of a detector click. In our setup, when there is no click during the exposition time (due either to the absence of a photon or to detector inefficiency), the test has to be repeated.

The main disadvantage of using pulsed lasers is that they have a mean frequency that depends on the working conditions of the laser, so each repetition of the experiment requires considerable time to stabilize the frequency. Their advantage is that they do not require any gates.

3. VIRTUAL-OR-REAL-PATH EXPERIMENT

As we saw in Section 2, there is an essential difference between our proposal and the one of Kwiat *et al.*[3,5] because their interaction-free experiment is based on repeated interrogations carried out by a real photon whereas ours is based on boundary conditions imposed on an empty photon path that contains no photon. Consequently, our approach does not give rise to the quantum Zeno effect, as to theirs does.[5] Nevertheless, in our experiment we can formulate a question whether the round-trip path that a photon sees when approaching the crystal is virtual or real. The question is similar to the virtual-or-real-photon question posed by Weinfurter *et al.*[6] and Fearn *et al.*[7] Both questions assume answers supported by classical formal reasoning and calculation. In the latter experiment mirrors suppress the propagation of downconverted waves forming standing waves by reflecting the waves back to the crystal. As soon as the mirrors were removed and instantaneously replaced by detectors, the detectors fired, triggered by the photons in the outgoing beams owing to the changed boundary condition. In our experiment, changing the boundary conditions, i.e., switching on the round-trip path, means allowing the round-trip path to wind up (according to the calculation presented in Fig. 3) even when there is no single photon in the path: the paths are real.

The experiment is presented in Fig. 4. It is a modification of the experiment shown in Fig. 1. We tune our FTIR–MOTTIR system to have as big a gap between the coupling prisms and the crystal as possible (e.g., corresponding to $R = 0.9999$). The Rochon prism is p rotated to match the phase shift fully as its O wave. Therefore, when the Pockels cell is off, the round-trip path is not influenced at all. When the Pockels cell is on, the path is redirected through Rochon prism p (as the E wave) into detector D_p. We switch on a cw laser and let it feed the system. When the Pockels cell is on, detector D_r should fire with the probability approaching 1. When it is off, detector D_t should fire with the probability approaching 1.

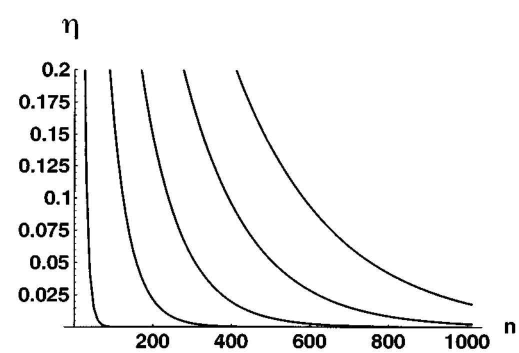

Fig. 3. Realistic values of η [ratio of the incoming and reflected powers, Eq. (9)] for $R = 0.98$, 0.99, 0.995, 0.997, and 0.998 (leftmost to rightmost curves) as functions of the number of round trips n.

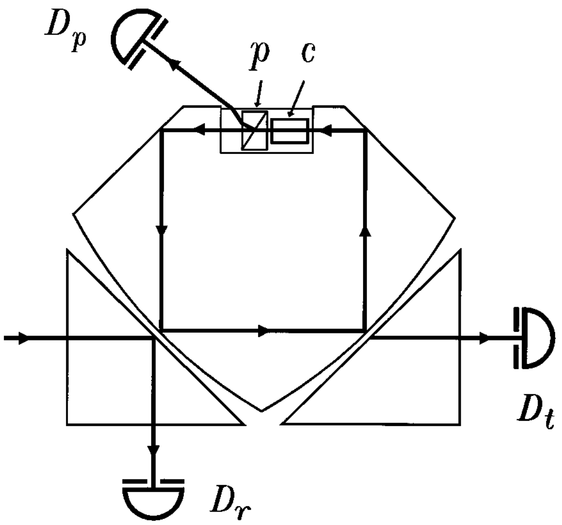

Fig. 4. The proposed virtual-or-real-path experiment. When Pockels cell c is on, it redirects the round-trip path through Rochon prism p into detector D_p, and therefore in most tests only detector D_r fires. When Pockels cell c is off, there is no influence on the round-trip path, and in most tests only detector D_t fires.

We carry out two kinds of measurement. The first kind of measurement involves switching the Pockels cell on and monitoring D_r immediately afterward. We adjust the intensity of the laser beam to have one photon in 0.1 ns on average. The fastest Pockels cells have reaction times as fast as 0.1 ns. The time that information traveling at the speed of light needs to spread from the Pockels cell to the incoming gap can be made as great as 4 ns by choice of the biggest available crystals. The fastest detectors have reaction times of less than 1 ns. Before we switch on the Pockels cell, practically only detector D_t fires. After we switch on the Pockels cell we monitor detector D_r and see whether it reacts instantaneously or after 4 ns.

The second kind of measurement consists of switching the Pockels cells from on to off and monitoring detector D_r immediately afterward. We lower the intensity of the laser beam to have one photon in 10 ns on average. We calculated that for $R = 0.9999$ the resonance is fully established after 100 ns; i.e., after that time D_r can barely fire. We monitor D_r within this 100 ns and see whether detector D_r stops firing immediately or only after it fires several times within the first 100 ns. We have chosen the intensity of the incoming beam to permit only one photon in 10 ns to make sure that, after the Pockels cell is switched off, only an empty wave is coming into the system following each subsequent photon. A variation on the experiment would be to lower the intensity of the laser beam further to less than 1 photon in 100 ns.

4. CONCLUSION

In summary, we have devised a feasible and, in principle, rather simple experimental scheme for interaction-free detection of an absorbing object (bomb) that rests on the classical behavior of a ring resonator into which light is fed. In case of resonance and in the absence of an obstacle within the cavity, a detector D_t in the exit channel will respond with high probability, whereas a detector D_r placed in the light beam reflected from the entrance mirror will almost never detect a photon. When an object is introduced into the cavity, the situation is completely reversed. The absence or presence of the object will in any case be indicated by a detector click. This is a great advantage of our scheme, because there is no need to ensure that a photon is actually impinging upon the object. Moreover, we suggest a modification of the experimental scheme that would allow us to measure time delays between blocking the resonator (or undoing a blocking) and the effect that this has on the detection probabilities. We apply the scheme to the Heisenberg microscope and the "Welcher Weg" experiment in Ref. 13.

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