THERE IS A FORMAL DIFFERENCE BETWEEN THE COPENHAGEN AND THE STATISTICAL INTERPRETATION OF QUANTUM MECHANICS

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ABSTRACT. The notion of repeatability of quantum spin measurements carried out on individual quantum systems is reconsidered. It is shown that a YES-NO measurement of the first kind of an arbitrary spin can be considered repeatable if and only if a particular physically meaningful function exhibits a jump-discontinuity for just one end point of a closed interval. The result serves to formulate a difference between the Copenhagen and the statistical interpretation **within** quantum formalism.

1. INTRODUCTION

In quantum mechanics an opinion prevails that "a pure state provides a complete and exhaustive description of an individual system (e.g., an electron),"[1] and that "the system ... does ... have a value for any observable ... for which the state is an eigenvector."[1] Such a view is supported by the Copenhagen interpretation. The view is, however, opposed by "the statistical interpretation, according to which a pure state provides a description of certain statistical properties of an ensemble of similarly prepared systems, but need not provide a complete description of an individual system."[1]

Arguments so far advanced in favour of one or the other view have been formulated outside the quantum formalism (e.g., by Ballentine,[1] Park,[2] and Margenau[3]) since it had been taken for granted that a difference between the interpretations could not be formulated within it.[4]

The purpose of this article is to formulate a difference between the views **within** the quantum formalism and to show that they do not coincide within it, although both are compatible with it.

To this end, an expression (given by Eq. (6)) is constructed which is a function of the relative frequency of measured data as well as of the corresponding theoretical probability and which has a well defined physical meaning. According to the Copenhagen interpretation the function exhibits a sudden jump at the end points of the closed interval $[0,1]$. According to the statistical interpretation the function is continuous on the whole interval. While a difference in experimental values of the function on the open interval $(0,1)$ as opposed to the closed interval $[0,1]$ cannot be expected to be measurable, an important physical contribution of this result is that the assumption of the repeatability of YES-NO measurements carried out on individual quantum systems implies an actual jump in the value of a well defined function for just one mathematical point of an interval.

As for the repeatability hypothesis, it has recently been shown that non-discrete observables do not satisfy it.[5] It has also been shown that in the presence of a conservation law even discrete observables are only approximately measurable.[6] Thus, both kinds of observables allow a value from their spectra to be a result of a measurement, but the value cannot be ascribed to a particular property of the measured individual system in a particular state. As opposed to this situation, when individual systems are subjected to YES-NO measurements of a discrete observable, unrestricted by any conservation law, the eigenvalue of the measured observable (projector) can always be taken to correspond to a particular property of the ensemble of individual systems. Thus, for repeated YES-NO measurements of an unrestricted discrete observable a YES-event occurs with certainty, i.e. with probability equal to unity, and from a statistical point of view such measurements can always be considered as repeatable. However, in looking at individual events we face the following dilemma.

We can take a view that a YES-event with probability one always occurs. In this case a measurement is considered repeatable in both senses: statistical and individual. An individual system is considered to have a particular property.

The other possibility is to assume that a YES-event with probability equal to unity need not always occur. In this case the repeatability is not admitted so far as individual events are concerned.

The latter view is attractive because it suggests a unification of descriptions of quantum measurements in the sense that the approximately repeatable measuring process might be a model of measurements in quantum mechanics, not only for continuous and restricted discrete observables, but also for unrestricted discrete observables. The "measure" for the (non)occurence of a YES-event with probability one provided in this paper will hopefully contribute to such an elaboration, e.g. in the recently proposed measurement statistics interpretation of quantum mechanics.[7)]

The paper is organized as follows. In Sec. 2 some properties of the relative frequency of individual events in Bernoulli trials are formulated in the form of lemmas. In Sec. 3 we define an expression, Eq. (6), which includes a difference between a frequency and the appropriate probability, but which does not vanish as the number of experiments approaches infinity. This expression enables us to consider the extreme values of probability, i.e. those equal to unity and zero, for which a distinction between the statistical and the Copenhagen interpretation can be expressed within the quantum formalism. A discussion of the obtained result is carried out in Sec. 4.

2. SOME PROPERTIES OF THE RELATIVE FREQUENCIES OF QUANTUM YES-NO EVENTS

The first basic feature of any quantum YES-NO measurement of the first kind is that particular individual events are completely independent. The second basic feature of such measurements is that trials form an exchangeable sequence. Taken together, the trials are Bernoulli trials, i.e. they form Bernoulli sequences. Thus we can estimate ideal (quantum) frequencies, i.e. frequencies of an infinite number of individual YES-NO experiments, by means of (quantum theoretical) probabilities as elaborated below.

Let us call the event $X_i = 1$ a success on the ith trial. The frequency with which the result 1 is obtained is

$$f = \frac{1}{N} \sum_{i=1}^{N} X_i = \frac{N_+}{N}$$

where N_+ is the number of successes in the first N trials.

The probability that the ith trial gives the result 1 (as given by the quantum theory) is denoted as $P(X_i = 1) \in [0,1]$. The expectation of the frequency is

$$\langle f \rangle = \frac{1}{N} \sum_{i=1}^{N} P(X_i = 1)$$

The exchangeability: $P(X_1 = 1) = \ldots = P(X_N = 1) =: p$ then gives $\langle f \rangle = p$.

The probability that the ith trial and the jth trial give the result 1 is $P(X_i = 1, X_j = 1) \in [0,1]$. Since the independence of individual events can be expressed as $P(X_i = 1, X_j = 1) = P(X_i = 1)P(X_j = 1) = p^2$, for $i \neq j$ and since $P(X_i = 1, X_i = 1) = p$, the expectation of the square of the frequency is

$$\langle f^2 \rangle = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} P(X_i = 1, X_j = 1) = \left(Np + (N^2 - N)p^2\right)/N^2$$

Thus the appropriate standard deviation is

$$\Delta p = \sqrt{(\langle f^2 \rangle - \langle f \rangle^2)} = \sqrt{p(1-p)/N} = \sqrt{pq/N} \tag{1}$$

where $q := 1 - p$.

By the Markov theorem[8] we obtain that the frequency $f = N_+/N$ converges to p "in probability" and by Lévy theorem[3] it then follows that $f = N_+/N$ converges to p almost certainly (a.c.), in symbols $N_+/N \xrightarrow{\text{a.c.}} p$, which means

$$P(\lim_{N\to\infty} \frac{N_+}{N} = p) = 1 \qquad (2)$$

(Note that N_+ is actually a function of probability and of the number of trials: $N_+ = N_+(p,N)$ and that the lemmas below apply to p from the open interval $0 < p < 1$.)

However, on which minimal set and in which way does the frequency N_+/N converge to p?

We answer this question in the form of the following lemmas.

Lemma 1. Let p, N_+, and N be defined as above. If $0 < p < 1$, $-\frac{1}{2} < k < -\frac{1}{3}$, $0 < \gamma < \infty$, and $0 < \eta < \infty$, then

$$\lim_{N\to\infty} P\left(p - \eta(pq)^{\gamma}N^{k} \le \frac{N_+}{N} \le p + \eta(pq)^{\gamma}N^{k}\right) = 1 .$$

Proof. As given in Ref. 9. □

Lemma 2. Let p, N_+, N, γ, and η be defined as above. If $n < -\frac{1}{2}$ then

$$\lim_{N\to\infty} P\left(p - \eta(pq)^{\gamma}N^{n} \le \frac{N_+}{N} \le p + \eta(pq)^{\gamma}N^{n}\right) = 0 .$$

Proof. As given in Ref. 9. □

Lemma 3. Let p, N_+, N, γ, and η be defined as above. Then

$$\lim_{N\to\infty} P\left(p - \eta(pq)^{\gamma}N^{-\frac{1}{2}} \le \frac{N_+}{N} \le p + \eta(pq)^{\gamma}N^{-\frac{1}{2}}\right) = \frac{1}{2\pi}\int_{-\varepsilon}^{\varepsilon} \exp(-\frac{x^2}{2})dx$$

where $\varepsilon = \eta(pq)^{\gamma-\frac{1}{2}}$.

Proof. As given in Ref. 9. □

The lemmas put together state that the convergence of the frequency N_+/N to probability p takes place on a set defined as

$$[p - \eta(pq)^{\gamma}N^{k}, p - \eta(pq)^{\gamma}N^{n}) \cup (p + \eta(pq)^{\gamma}N^{n}, p + \eta(pq)^{\gamma}N^{k}]$$

where k and n (specified above) are arbitrarily close to $-\frac{1}{2}$. This means that a frequency N_+/N which converges to an appropriate probability p is "improperly" Gaussian-like distributed so far as only arbitrary large N's, instead of the ones approaching infinity, are considered; i.e. an arbitrary narrow strip is "cut out" from the middle of the Gaussian. (The lemmas remain "almost unchanged" when N is only "sufficiently large" instead of approaching infinity.)

When N tends to infinity we can hardly speak about a distribution, because $(pq)^{\gamma}N^k \to 0$, but we can express the result by saying that N_+/N acquires a value which is strictly equal to p by probability zero:

$$\lim_{N\to\infty} P\left(\frac{N_+}{N} = p\right) = 0 \tag{3}$$

The equation shows that the values of the frequency never cluster strictly at p but only around p, and puts forward an interesting characterization of the stochasticity of frequencies of the Bernoulli trials. In this case, the above statement that N_+/N is not "properly" Gaussian-like distributed means that a line centered at p is "cut out" from an infinitely narrow strip to which the Gaussian "shrinks."

Let us now consider the particular cases of probability equal to unity. Theorems concerning Bernoulli trials, our lemmas included, do not say anything about the strict value $p = 1$. It follows from Lemma 1 that $N_+(p,N)/N \to 1$ when $p \to 1$ and $N \to \infty$. Thus we are tempted to expect to have strictly $N_+(1,N) = N$. However, as we show in the next section, so far as the quantum YES-NO measurements are concerned, we should be tempted to expect just the opposite.

3. SPIN MEASUREMENTS ON INDIVIDUAL QUANTUM SYSTEMS

Let us consider spin preparation-detection measurements for spin-s particles. Quantum systems are prepared, one by one, by a preparation device (a Stern-Gerlach device) and detected, one by one, by a detection device (another Stern-Gerlach device) deflected at an angle α relative to the preparation. In a word, we carry out quantum YES-NO measurements on individual systems. Quantum mechanics then predicts that the relative frequency N_+/N of the number N_+ of detections of the prepared property (spin projection m - prepared in the statistical sense of the word) on the systems among the total number N of the prepared systems, approaches probability $p = p^{(s)}_{mm}(\alpha) = (d^{s}_{mm}(\alpha))^2$ (where $d^{s}_{mm}(\alpha)$ is a diagonal element of the rotation matrix) as N approaches infinity. Since $(d^{s}_{-m-m}(\alpha))^2 = (d^{s}_{mm}(\alpha))^2$ in the sequel we shall consider only $m \geq 0$. Diagonal elements of the rotation matrix are in this case defined as follows:[10)]

$$d^{s}_{mm}(\alpha) = (s+m)!(s-m)! \sum_{k=0}^{s-m} \frac{(-1)^k}{(s+m-k)!(s-m-k)!(k!)^2} (\cos\frac{\alpha}{2})^{2s-2k} (\sin\frac{\alpha}{2})^{2k} . \quad (4)$$

By means of Jacobi polynomials[11)] this can be expressed as

$$d^{s}_{mm}(\alpha) = (\cos\frac{\alpha}{2})^{2m} P^{(0,2m)}_{s-m}(\cos\alpha).$$

From the functional form of $d^{s}_{mm}(\alpha)$ as given by Eq. (4) it is obvious that there is always such $\alpha = \alpha_1$ that $p = (d^{s}_{mm}(\alpha))^2$ is a continuous monotonic decreasing function defined on $[0,\alpha_1]$ and differentiable on $(0,\alpha_1)$. Hence it is invertible and its inversion function $\alpha = \alpha^{s}_{mm}(p) = \alpha(p)$ is also differentiable - on $(p_1, 1)$ where $p_1 = p(\alpha_1)$ - and $d\alpha/dp = (dp/d\alpha)^{-1}$ holds.[12)]

Therefore, in the definition of the absolute value of the derivative

$$\left|\frac{d\alpha}{dp}\right| = \lim_{h\to 0} \left|\frac{\alpha(p+h) - \alpha(p)}{h}\right| = \left|\frac{dp}{d\alpha}\right|^{-1} \quad (5)$$

we are allowed to replace h by a particular net or series converging to zero and we shall do so using a particular expression, which will eventually serve to ascribe values of the frequency N_+/N to p+h and to obtain the function (6), which includes a difference between the frequency and the appropriate probability but which does not vanish as the number of experiments approaches infinity. This function will then serve us as a "measure" of difference between the statistical and the Copenhagen interpretation of the quantum formalism.

As a candidate for the above-mentioned expression which is to be substituted for h in Eq. (5) the lemmas from Sec. 2 single out

$\chi(N)(pq)^{\gamma}N^{\beta}$, where $0<|\chi(N)|<\infty$, $0<\gamma<\infty$, and $-\infty<\beta<0$. Let us proceed along this line.

The points $p\pm|\chi(N)|(pq)^{\gamma}N^{\beta}$ are the end points of the interval $[p-|\chi(N)|(pq)^{\gamma}N^{\beta}, p+|\chi(N)|(pq)^{\gamma}N^{\beta}]$ and, according to Lemmas 1 and 2, we can accomplish the ascription of $\chi(N)(pq)^{\gamma}N^{\beta}$ to h in (5) only when $\beta=-\frac{1}{2}$. For, according to Lemma 2, the frequency N_+/N almost never acquires a value on the interval when the latter is defined by $\beta<-\frac{1}{2}$ and, according to Lemma 1, the frequency N_+/N has almost all values on the interval, but perhaps none at its end points when the interval is defined by $\beta>-\frac{1}{2}$.

On the other hand, according to Lemma 3, for $\gamma>\frac{1}{2}$ the frequency N_+/N finds its values on the interval $[p-\eta(pq)^{\gamma}N^{-\frac{1}{2}}, p+\eta(pq)^{\gamma}N^{-\frac{1}{2}}]$ with a probability which is arbitrary close to zero, no matter which confidence coefficient η is chosen, when p approaches unity - and this is the case in which we are interested. When $\gamma<\frac{1}{2}$ and p approaches unity the frequency again has almost all values on the interval, but perhaps none at its end points.

Thus, we are left with $\beta=-\frac{1}{2}$ and $\gamma=\frac{1}{2}$, our points boiling down to $p+\chi(N)\Delta p$, where Δp is given by Eq. (1). In other words, only for these values of β and γ is the convergence of N_+/N to p assured (on the minimal set from Sec. 2).

According to Lemma 3, for every N, for $\eta\geq|\chi(N)|$ large enough, almost all (i.e. with a probability approaching one) values of N_+/N, as well as of $p+\chi(N)\Delta p$, lie within the interval $[p-\eta\Delta p, p+\eta\Delta p]$. On the other hand, for $\eta'\leq|\chi(N)|$ small enough, almost all values of N_+/N as well as of $p+\chi(N)\Delta p$, lie outside the interval $[p-\eta'\Delta p, p+\eta'\Delta p]$. Besides, according to Lemma 2 and Eq. (3) the probability of N_+/N being strictly equal to p is zero.

Hence, for every N there is almost certainly such $\chi(N)$, so as to make $p+\chi(N)\Delta p$ equal to N_+/N and $\chi(N)\Delta p$ converging to zero (without making it strictly equal to zero) as N approaches infinity.

In Eq. (5) we can, therefore, choose $h=\chi(N)\Delta p$ and substitute N_+/N for $p+h$, thus obtaining

$$\left|\frac{d\alpha}{dp}\right| = \lim_{N\to\infty} \frac{|\alpha(N_+/N) - \alpha(p)|N^{\frac{1}{2}}}{|\chi(N)|\sqrt{pq}} = \left|\frac{dp}{d\alpha}\right|^{-1}$$

After a rearrangement it gives (for $p_1 < p < 1$)

$$G(p) := L^{-1} \lim_{N\to\infty} \left(|\alpha(N_+/N) - \alpha(p)|N^{\frac{1}{2}}\right) = \sqrt{pq}\left|\frac{dp}{d\alpha}\right|^{-1} =: H(p) \qquad (6)$$

where $L = \lim_{N\to\infty}|\chi(N)|$ is a bounded ($0 < L < \infty$) random (stochastic) variable which does not have a definite value, in the same way in which, e.g., $-1 \leq \lim_{x\to\infty} \sin x \leq 1$ does not have a definite value. However, the randomness of L is matched by the randomness of the other limit in Eq. (6), so that G(p) turns out to be a continuous nonvanishing function on $(p_1, 1)$. The last statement can receive the following elaboration.

From Eq. (6), for $p_1 < p < 1$, where p is defined as above, we get

$$G(p) = H(p) = \frac{\sqrt{(1-d^s_{mm})(1+d^s_{mm})}}{2\left|\frac{dd^s_{mm}}{d\alpha}\right|}$$

The derivation from the denominator can be written as

$$\frac{dd^s_{mm}}{d\alpha} = -2^{-m}\sin\alpha \frac{d\left((1+\cos\alpha)^m P^{(0,2m)}_{s-m}(\cos\alpha)\right)}{d(\cos\alpha)}$$

which together with[11)]

$$\frac{d}{dx} P^{(0,2m)}_{s-m}(x) = \frac{1}{2}(s+m+1)\, P^{(1,2m+1)}_{s-m-1}(x)$$

gives

$$\frac{dd^s_{mm}(\alpha)}{d\alpha} = -\sin\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}\right)^{2m-1}\left(mP^{(0,2m)}_{s-m}(\cos\alpha) + (s+m+1)\cos^2\frac{\alpha}{2}\, P^{(1,2m+1)}_{s-m-1}(\cos\alpha)\right)$$

By using the expression (4) for $d^s_{mm}(\alpha)$ and writing explicitly the terms for $k = 0$ and $k = 1$ the expression $1 - d^s_{mm}$ can be written as follows

$$1-(\cos\frac{\alpha}{2})^{2s}+(s^2-m^2)(\cos\frac{\alpha}{2})^{2s-1}(\sin\frac{\alpha}{2})^2-\cdots$$

where the dots represent terms containing higher powers of $\sin\frac{\alpha}{2}$. With the help of $\cos 2\beta = 1-2\sin^2\beta$ and $\sin 2\beta = 2\sin\beta\cos\beta$ one obtains

$$1-d^s_{mm}(\alpha) = 4s\sin^2\frac{\alpha}{4}-\cdots+4(s^2-m^2)\sin^2\frac{\alpha}{4}\cos^2\frac{\alpha}{4}(\cos\frac{\alpha}{2})^{2s-1}-\cdots$$

where the dots denote terms containing higher powers of $\sin\frac{\alpha}{4}$.

Taken together we obtain

$$H(p) = \frac{\sin\frac{\alpha}{4}\left\{\left(s+(s^2-m^2)\cos^2\frac{\alpha}{4}(\cos\frac{\alpha}{4})^{2s-1}+\cdots\right)\left(1+(\cos\frac{\alpha}{2})^{2m}P^{(0,2m)}_{s-m}(\cos\alpha)\right)\right\}^{\frac{1}{2}}}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}(\cos\frac{\alpha}{2})^{2m-1}\left(mP^{(0,2m)}_{s-m}(\cos\alpha)+(s+m+1)\cos^2\frac{\alpha}{2}P^{(1,2m+1)}_{s-m-1}(\cos\alpha)\right)}$$

where the dots denote terms containing $\sin\alpha$ in the numerator.

Since [11] $P^{(\mu,\nu)}_n(1) = \frac{(n+\mu)!}{n!\mu!}$, for $s>m$ we get

$$\lim_{p\to 1} G(p) = \lim_{p\to 1} H(p) = \lim_{\alpha\to 0} H\left(p(\alpha)\right) = 2^{-\frac{1}{2}}(s^2+s-m^2)^{-\frac{1}{2}} \qquad (7)$$

For $s=m$ we get $\lim_{p\to 1} H(p) = (2s)^{-\frac{1}{2}}$. Since the former expression (7) boils down to the latter for $s=m$ and since $\left(d^s_{-m-m}(\alpha)\right)^2 = \left(d^s_{mm}(\alpha)\right)^2$ we finally obtain that the expression (7) holds for $m=-s,\ldots,+s$.

Turning our attention to the probability equal to one we see from the obtained expression for $H(p)$ that H is not defined for the probability equal to one: $H(1) = \frac{0}{0}$. However, its limit exists and is given by expression (7). Thus a continuous extension of H to $(p_1,1]$ exists and is given by $\tilde{H}$, where $\tilde{H}(p) := H(p)$ for $p\in(p_1,1)$ and $\tilde{H}(1)$ is equal to the right-hand side of Eq. (7).

The function G, on the other hand, cannot be approached in the same way because we do not know whether $G(1)$ is well defined and if it is, we do not know which values it should be ascribed. Namely, the statistical and the Copenhagen interpretation differ in this point, the latter demanding $G(1)=0$. Thus we are left with the following three possibilities:

1. $G(p)$ is continuous at 1. A necessary and sufficient condition for that is $G(1) = \lim_{p\to 1} G(p)$. In this case we cannot strictly have $N_+(1,N)=N$ since then $G(1)=0\neq\lim_{p\to 1} G(p)$ obtains a contradiction.

2. $G(1)$ is undefined. In this case we also cannot have $N_+(1,N) = N$ since the latter equation makes $G(1)$ defined, i.e. equal to zero.

3. $G(1) = 0$. In this case we must have $N_+(1,N) = N$. And vice versa: if the latter equation holds we get $G(1) = 0$.

Hence, a YES-NO measurement of a discrete spin observable s can be considered repeatable with respect to individual measured systems, if and only if $G(p^s_{mm})$, defined by Eq. (6), exhibits a jump-discontinuity for $p^s_{mm} = 1$ in the sense of point 3 above.

4. DISCUSSION

The result we obtained in the previous section states that the difference $|\alpha(N_+/N) - \alpha(p)|$ between the angle $\alpha(N_+/N)$, measured by "counting" individual detections, and the angle $\alpha(p)$, measured by a macroscopic device, multiplied by $N^{\frac{1}{2}}$, i.e. the expression $|\alpha(N_+/N) - \alpha(p)|N^{\frac{1}{2}}$, never vanishes as N approaches infinity.

For the strict value $p = 1$ we, however, face a dilemma.

Shall we assume that the difference in question suddenly drops to zero for p being strictly equal to unity, thus adopting the Copenhagen interpretation of quantum mechanics and the repeatability of YES-NO measurements?

Or shall we assume the continuity of $G(p)$ and the validity of Eq. (3) for $p = 1$ as well, thus adopting the statistical interpretation and banishing repeatable measurements on individual systems from quantum mechanics altogether?

By adopting the former interpretation we cannot but assume that nature differentiates open intervals from closed ones, i.e. distinguishes between two infinitely close points. One could try to object that this consequence of the obtained result is overemphasized and try to argue that for α's which are infinitely close to zero, i.e. for p's infinitely close to unity, the expression $|\alpha(N_+/N) - \alpha(N)|N^{\frac{1}{2}}$ approaches zero for an arbitrary large N. However, such an argument does not hold water since $p = \langle\frac{N_+}{N}\rangle$ and whenever p is not strictly equal to one, N_+ - being an integer - cannot be greater than $N - 1$. Therefore, p simply cannot be left infinitely close to 1 without first N being infinitely large. And we have shown in Sec. 3 that $\lim_{p\to 1}\lim_{N\to\infty}(|\alpha(N_+/N)-\alpha(p)|N)\neq 0$. N and p cannot simultaneously tend to infinity and one, respectively, either, since the existence of such a simultaneous limit implies the existence and equality of the appropriate successive limits. As one of the successive limits turned out not to exist, a simultaneous limit cannot exist either.

By adopting the latter interpretation, we allow but a bare statistical meaning to quantum probabilities, including those ones which are equal to unity and zero, and deny them a unique experimental meaning with which all individual systems would comply.

Since the consideration of an experiment which would differentiate between the two interpretations is rather beyond the scope of the present paper, we shall close the discussion by comparing the obtained result with a possible classical one.

Had, e.g., electrons obeyed a classical "linear" probability law, the deflection angle under consideration would have been expressible as $\alpha=\pi(1-p)$.[13] In this case Eq. (6) gives $G(p)=\pi\sqrt{pq}$, which is, as opposed to the quantum case, well defined for $p\in[0,1]$ and we meet no problems for $p=1$. Thus, for a classical probability the assumption $N_+(1,N)=N$ invokes no dilemma since $G(1)=H(1)=0$.

Now, for spins high enough we should expect quantum objects to behave almost classically. And really, our result provides us with such a limit since from the expression (7) we obtain

$$\lim_{s\to\infty}\lim_{p\to 1} G(p^s_{mm}) = 0.$$

* * *

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