Mladen PAVIČIĆ, Department of Mathematics, University of Zagreb, Rakušina 1, Poštanski pretinac 165, YU-41001 Zagreb, Yugoslavia

PROBABILISTIC SEMANTICS FOR QUANTUM LOGIC

A probabilistic semantics for quantum logic is formulated by means of an ultrastrongly ordering probability function. The soundness is proved as well as the completeness, with the help of a plausible transition function.

Recently, probabilistic semantics for a number of logics were formulated as substitutes for Krike's semantics. Thus, Leblanc [1] formulated a probabilistic semantics for first-order logic; Morgan [2] formulated a probabilistic semantics for every extension of propositional logic, modal logic included [3]; Morgan and Leblanc [4], and van Fraassen [5] formulated probabilistic semantics for intuitionistic logic.

As for quantum logic a formulation of its probabilistic semantics is more than a mere substitute for Kripke's model. For Goldblatt [6] proved that there exists no condition of the first order a possible accessibility relation could be subjected to for quantum logic, and therefore that at any case no simple frame for Kripke's semantics exists.

Since, however, orthologic [7] (minimal quantum logic [8]) does have Kripke's semantics we formulated quantum logic starting from orthologic in order to stress a particular extension which converts orthologic into quantum logic.

Following [7] we adopted Ackermann's schemata to formulate quantum logic as well as orthologic. We define quantum logic (orthologic) as a system which contains the following axioms and rules of inferences (the same, with the exception of the last rule of inference) for A, B,.. (propositions) from the set of propositions, **Q**:

Axioms:		Rules of inference:	
A1:	$A \dashv\vdash A$	R1:	$A \vdash B \;\;\&\;\; B \vdash C \;\Rightarrow\; A \vdash C$
A2:	$A \dashv\vdash \neg\neg A$	R2:	$A \vdash B \;\Rightarrow\; \neg B \vdash \neg A$
A3:	$A \wedge B \vdash A, \quad A \wedge B \vdash B$	R3:	$A \vdash B \;\;\&\;\; A \vdash C \;\Rightarrow\; A \vdash B \wedge C$
A4:	$A \wedge \neg A \vdash B$	R4:	$A \vdash B \;\;\&\;\; \neg A \wedge B \vdash C \wedge \neg C \;\Rightarrow\; B \vdash A$,

where a scheme $A \vdash B$ is a sequence of two propositions, A and B, and '$\vdash$' is regarded as a sign of logical entailment and can appear only once in an expression of the object language, i.e. cannot be nested or iterated. In general, we define logical entailment in quantum logic (orthologic) as follows:

Definition Let G be a non-empty set of propositions from Q. A proposition B is said to be Q-derivable (in symbols: $G \vdash B$, which reads "G entails B") if there exist: $B_1, B_2, \ldots, B_n \in G$ such that $B_1 \wedge B_2 \wedge \ldots \wedge B_n \vdash B$.

Before passing to the afore-mentioned formulation of quantum logic with the help of orthologic we shall define the possible implications in both logics:

Definitions of the implications in orthologic: $A \rightarrow_0 B := \neg A \vee B$ ("classical");

$A \rightarrow_1 B := \neg A \vee (A \wedge B)$ ("Sasaki", i.e. "Mittelstaedt", i.e. "ortho-");
$A \rightarrow_2 B := B \vee (\neg A \wedge \neg B)$ ("Dishkant");
$A \rightarrow_3 B := (A \wedge B) \vee (\neg A \wedge B) \vee (\neg A \wedge \neg B)$ ("relevance");
$A \rightarrow_4 B := (A \wedge B) \vee (\neg A \wedge B) \vee ((\neg A \vee B) \wedge \neg B)$ (./.);
$A \rightarrow_5 B := (\neg A \wedge \neg B) \vee (\neg A \wedge B) \vee ((\neg A \vee B) \wedge A)$ ("Kalmbach").

Thereupon, we define a "minimal criterion for a connection between the logical entailment and the above implications as I(i):

<u>Definition</u> I(i): $A \vdash B \Longleftrightarrow C \vee \neg C \vdash A \rightarrow_i B$, $i = 0, 1, .., 5$.

Now, the formulation of quantum logic by means of orthologic follows from:

<u>Theorem</u> Quantum logic := [A1-A4 & R1-R4] $\Longleftrightarrow$ [A1-A4 & R1-R3 & I(i)] , i = 1,..,5.
(Remark: Classical logic $\Longleftrightarrow$ [A1-A4 & R1-R3 & I(0)] := [orthologic & I(0)])

Given the last theorem we see that quantum logic is nothing but orthologic extended just so as to make '$A \rightarrow_i B$', i=1,..,5, a logical truth iff '$A \vdash B$'. Making the extension, we loose the possibility to construct Kripke's frame of the first order, which orthologic has, and a question arises as to whether we should complicate quantum logic, as well as its probabilistic semantics which is still at hand, trying to single out one of the five possible implications, or not. In our opinion, we should not. For any such attempt seems to rule out a possibility for a transition probability simple enough to correspond to individual YES-NO measurements. Therefore, we propose the implication be "defined" by I(i) thus merging all the five implications into one entailment.

The probabilistic semantics, PQ of quantum logic, QL is given by means of a quantum probability function Pr: QL $\rightsquigarrow$ [0,1] which meets the conditions P1-P7, given below.

We call $A \in Q$ Pr-normal if there is at least one proposition $B \in Q$ such that $Pr(B|A) \neq 1$, and Pr-abnormal if there is no such proposition.

The constraints which every Pr function meets, for all $A, B, .. \in Q$, are:

P1: $0 \leqslant Pr(B|A) \leqslant 1$
P2: $Pr(A|A) = Pr(A|A \wedge B) = Pr(B|A \wedge B) = Pr(\neg B \vee A|A) = 1$
P3: $Pr(\neg B|A) = 1$ if B is Pr-abnormal
P4: $Pr(B \wedge \neg B|A) = 0$ if A is Pr-normal
P5: $Pr(B|A) = 1$ & $Pr(C|A) = 1 \Longrightarrow Pr(B \wedge C|A) = 1$
P6: $Pr(\neg B|A) + Pr(B|A) = 1$ if A is Pr-normal
P7: $Pr(B_i|B_j) = 0,\ \forall i \neq j \Longrightarrow Pr(\bigvee_i B_i|A) = \sum_i Pr(B_i\ A)$ if A is Pr-normal.

<u>Definition</u> Let G be a non-empty set of propositions from Q. A proposition B is said to be Q-probabilistically derivable from G (in symbols: $G \models B$, which reads: "G probabilistically entails B") if, for a function Pr: QL $\rightsquigarrow$ [0,1], which meets the constraints P1-P7, there exist $B_1, .., B_n \in G$ such that $Pr(B|B_1 \wedge .. \wedge B_n) = 1$. A scheme $A \vdash B$ is said to be probabilistically valid if $Pr(B|A) = 1$.

In order to prove the completeness we used a plausible transition probability $Pr(A \rightsquigarrow B)$ to define a probability function Pr of PQ.

<u>Definition</u>

$$Pr(B|A) = \begin{cases} 1 & \Longleftarrow A \vdash B \\ Pr(A \rightsquigarrow B) & \Longleftarrow A \nvdash B , \end{cases}$$

where $Pr(A \rightsquigarrow B)$ is a transition probability which satisfies the following conditions:

1. $A \nvdash B \;\&\; A \nvdash \neg B \Longrightarrow 0 < Pr(A \rightsquigarrow B) < 1$
2. $A \nvdash B \;\&\; A \vdash \neg B \Longrightarrow Pr(A \rightsquigarrow B) = 0$
3. $A \nvdash B \;\&\; A \nvdash \neg B \Longrightarrow Pr(A \rightsquigarrow B) + Pr(A \rightsquigarrow \neg B) = 1$
4. $A \nvdash B_i \;\&\; B_j \vdash \neg B_i,\ \forall i \neq j \Longrightarrow Pr(A \rightsquigarrow \bigvee_i B_i) = \sum_i Pr(A \rightsquigarrow B_i)$.

For the semantical system, PQ we are able to prove:

Theorem $\quad G \vdash A \Longleftrightarrow G \models A$

which is our main result.

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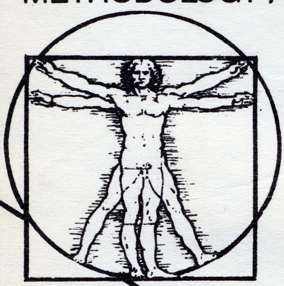

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