

Section 8

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PROBABILISTIC SEMANTICS FOR QUANTUM LOGIC

A probabilistic semantics for quantum logic is formulated by means of an ultrastrongly ordering probability function. The soundness is proved as well as the completeness, with the help of a plausible transition function.

Recently, probabilistic semantics for a number of logics were formulated as substitutes for Kripke's semantics. Thus, Leblanc [1] formulated a probabilistic semantics for first-order logic; Morgan [2] formulated a probabilistic semantics for every extension of propositional logic, modal logic included [3]; Morgan and Leblanc [4], and van Fraassen [5] formulated probabilistic semantics for intuitionistic logic.

As for quantum logic a formulation of its probabilistic semantics is more than a mere substitute for Kripke's model. For Goldblatt [6] proved that there exists no condition of the first order a possible accessibility relation could be subjected to for quantum logic, and therefore that at any case no simple frame for Kripke's semantics exists.

Since, however, orthologic [7] (minimal quantum logic [8]) does have Kripke's semantics we formulated quantum logic starting from orthologic in order to stress a particular extension which converts orthologic into quantum logic.

Following [7] we adopted Ackermann's schemata to formulate quantum logic as well as orthologic. We define quantum logic (orthologic) as a system which contains the following axioms and rules of inferences (the same, with the exception of the last rule of inference) for A, B, \dots (propositions) from the set of propositions, Q :

Axioms:

- A1: $A \dashv\vdash A$
- A2: $A \dashv\vdash \neg\neg A$
- A3: $A \wedge B \dashv\vdash A, \quad A \wedge B \dashv\vdash B$
- A4: $A \wedge \neg A \dashv\vdash B$

Rules of inference:

- R1: $A \vdash B \ \& \ B \vdash C \Rightarrow A \vdash C$
- R2: $A \vdash B \Rightarrow \neg B \vdash \neg A$
- R3: $A \vdash B \ \& \ A \vdash C \Rightarrow A \vdash B \wedge C$
- R4: $A \vdash B \ \& \ \neg A \wedge B \vdash C \wedge \neg C \Rightarrow B \vdash A$

where a scheme $A \vdash B$ is a sequence of two propositions, A and B , and ' \vdash ' is regarded as a sign of logical entailment and can appear only once in an expression of the object language, i.e. cannot be nested or iterated. In general, we define logical entailment in quantum logic (orthologic) as follows:

Definition Let G be a non-empty set of propositions from Q . A proposition B is said to be Q -derivable (in symbols: $G \vdash B$, which reads " G entails B ") if there exist: $B_1, B_2, \dots, B_n \in G$ such that $B_1 \wedge B_2 \wedge \dots \wedge B_n \vdash B$.

Before passing to the afore-mentioned formulation of quantum logic with the help of orthologic we shall define the possible implications in both logics:

Definitions of the implications in orthologic: $A \rightarrow B := \neg A \vee B$ ("classical");

$A \xrightarrow{1} B := \neg AV(AAB)$ ("Sasaki", i.e. "Mittelstaedt", i.e. "ortho-");

$A \xrightarrow{2} B := BV(\neg A \wedge \neg B)$ ("Dishkant");

$A \xrightarrow{3} B := (A \wedge B) \vee (\neg A \wedge B) \vee (\neg A \wedge \neg B)$ ("relevance");

$A \xrightarrow{4} B := (A \wedge B) \vee (\neg A \wedge B) \vee ((\neg AVB) \wedge \neg B)$ (./.);

$A \xrightarrow{5} B := (\neg A \wedge \neg B) \vee (\neg A \wedge B) \vee ((\neg AVB) \wedge A)$ ("Kalmbach").

Thereupon, we define a "minimal criterion for a connection between the logical entailment and the above implications as $I(i)$:

Definition $I(i)$: $A \vdash B \iff C \vee \neg C \vdash A \xrightarrow{i} B$, $i = 0, 1, \dots, 5$.

Now, the formulation of quantum logic by means of orthologic follows from:

Theorem Quantum logic := $[A1-A4 \ \& \ R1-R4] \iff [A1-A4 \ \& \ R1-R3 \ \& \ I(i)]$, $i = 1, \dots, 5$.

(Remark: Classical logic $\iff [A1-A4 \ \& \ R1-R3 \ \& \ I(\hat{0})] := [\text{orthologic} \ \& \ I(\hat{0})]$)

Given the last theorem we see that quantum logic is nothing but orthologic extended just so as to make ' $A \xrightarrow{i} B$ ', $i=1, \dots, 5$, a logical truth iff ' $A \vdash B$ '.

Making the extension, we loose the possibility to construct Kripke's frame of the first order, which orthologic has, and a question arises as to whether we should complicate quantum logic, as well as its probabilistic semantics which is still at hand, trying to single out one of the five possible implications, or not. In our opinion, we should not. For any such attempt seems to rule out a possibility for a transition probability simple enough to correspond to individual YES-NO measurements. Therefore, we propose the implication be "defined" by $I(i)$ thus merging all the five implications into one entailment.

The probabilistic semantics, PQ of quantum logic, QL is given by means of a quantum probability function $Pr: QL \rightsquigarrow [0,1]$ which meets the conditions P1-P7, given below.

We call $A \in Q$ Pr-normal if there is at least one proposition $B \in Q$ such that $Pr(B|A) \neq 1$, and Pr-abnormal if there is no such proposition.

The constraints which every Pr function meets, for all $A, B, \dots \in Q$, are:

P1: $0 \leq Pr(B|A) \leq 1$

P2: $Pr(A|A) = Pr(A|A \wedge B) = Pr(B|A \wedge B) = Pr(\neg B \vee A|A) = 1$

P3: $Pr(\neg B|A) = 1$ if B is Pr-abnormal

P4: $Pr(B \wedge \neg B|A) = 0$ if A is Pr-normal

P5: $Pr(B|A) = 1$ & $Pr(C|A) = 1 \implies Pr(B \wedge C|A) = 1$

P6: $Pr(\neg B|A) + Pr(B|A) = 1$ if A is Pr-normal

P7: $Pr(B_i|B_j) = 0$, $\forall i \neq j \implies Pr(\bigvee_i B_i|A) = \sum_i Pr(B_i|A)$ if A is Pr-normal.

Definition Let G be a non-empty set of propositions from Q. A proposition B is said to be Q-probabilistically derivable from G (in symbols: $G \models B$, which reads: "G probabilistically entails B") if, for a function $Pr: QL \rightsquigarrow [0,1]$, which meets the constraints P1-P7, there exist $B_1, \dots, B_n \in G$ such that $Pr(B|B_1 \wedge \dots \wedge B_n) = 1$. A scheme $A \vdash B$ is said to be probabilistically valid if $Pr(B|A) = 1$.

In order to prove the completeness we used a plausible transition probability $Pr(A \rightsquigarrow B)$ to define a probability function Pr of PQ.

Definition

$$Pr(B|A) = \begin{cases} 1 & \iff A \vdash B \\ Pr(A \rightsquigarrow B) & \iff A \not\vdash B \end{cases}$$

where $Pr(A \rightsquigarrow B)$ is a transition probability which satisfies the following conditions:

1. $A \not\vdash B \ \& \ A \not\vdash \neg B \Rightarrow 0 < \Pr(A \rightsquigarrow B) < 1$
2. $A \not\vdash B \ \& \ A \vdash \neg B \Rightarrow \Pr(A \rightsquigarrow B) = 0$
3. $A \not\vdash B \ \& \ A \not\vdash \neg B \Rightarrow \Pr(A \rightsquigarrow B) + \Pr(A \rightsquigarrow \neg B) = 1$
4. $A \not\vdash B_i \ \& \ B_j \vdash \neg B_i, \ \forall i \neq j \Rightarrow \Pr(A \rightsquigarrow \bigvee_i B_i) = \sum_i \Pr(A \rightsquigarrow B_i)$.

For the semantical system, PQ we are able to prove:

Theorem $G \vdash A \iff G \models A$

which is our main result.

References

- [1] Leblanc H., Probabilistic Semantics for First-Order Logic, *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 25 (1979), 497-509.
- [2] Morgan C.G., There Is a Probabilistic Semantics for Every Extension of Classical Sentences Logic, *Journal of Philosophical Logic*, 11 (1982), 431-442.
- [3] Morgan C.G., Simple Propositional Semantics for Propositional K, T, B, S4, and S5, *Journal of Philosophical Logic*, 11 (1982), 443-458.
- [4] Morgan C.G. and Leblanc H., Probabilistic Semantics for Intuitionistic Logic, *Notre Dame Journal of Formal Logic*, 42 (1983), 161-180.
- [5] van Fraassen B.C., Probabilistic Semantics Objectified: I. Postulates and Logics, *Journal of Philosophical Logic*, 10 (1981), 371-394.
- [6] Goldblatt, R., Orthomodularity Is Not Elementary, *The Journal of Symbolic Logic*, 49 (1984), 401-404.
- [7] Goldblatt R., Semantic Analysis of Orthologic, *Journal of Philosophical Logic*, 3 (1974), 19-35.
- [8] Dalla Chiara M.L., Quantum Logic, in: Gabbay D.M. and Guentner F. (Eds.), *Handbook of Philosophical Logic, Volume III: Alternatives to Classical Logic*, D. Reidel Publishing Comp., Dordrecht, Holland, 1986, p. 427.

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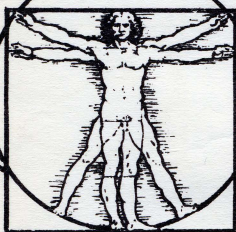
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8 МЕЖДУНАРОДНЫЙ КОНГРЕСС ПО ЛОГИКЕ,
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AN AXIOMATIC BASIS AS THE DESIRED FORM OF A PHYSICAL THEORY

Desired for what? Not for the mere practice of calculating
special problems. An axiomatic basis is necessary for discussing
fundamental problems as the interpretation of a theory, the phys-
ical meaning of its laws of nature, the logic of its interesting
language, the meaning of reality and possibility.

A physical theory T is composed of a mathematical theory M ,
a correspondence rule $(-)$ and a set of axioms A .

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The correspondence rule $(-)$ is a prescription of how to
translate into M each statement S in T in order to
obtain a statement S' in M . This translation is done by
means of devices, or rules, or axioms, or axioms in the
language of M . The translation is done by means of
axioms, or rules, or axioms in the language of M .

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under consideration. This does not mean that we use no theory at
all. But we may use only "pre-theories" $(-)$ to describe the funda-
mental domain D , that is theories already established before in-
terpreting the theory T to be considered.

The correspondence rules have the following form $(-)$:

Some facts in T are denoted by signs, say letters a, b, c, \dots .
In M some facts are denoted by pictorial signs $R_1, R_2, R_3, \dots, R_n$.
and some relations $R_1, R_2, R_3, \dots, R_n$ as pictorial relations. By virtue
of the correspondence rules, the facts of D are translated into an
additional text in M (sometimes called "observational report"),
which is of the form

$$\{ \dots \} \quad \{ \dots \} \quad \{ \dots \}$$

Here the a_i are real numbers which may be absent in some of
the relations R_i .

The correspondence rules are just rules for the translation
of propositions from common language or from the language of pre-
theories into the relations $(-)$.
There arise several questions:

1. How is the fundamental domain determined?