THE OTHER WAY ROUND: QUANTUM LOGIC AS METALOGIC

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One of the main dicta on natural sciences is that they should try to ascertain how things are and not why they are as they are; i.e. to describe – not to explain. Thus, a creed has been developed that theories of natural phenomena be analysed syntactically—not semantically. However, it is just the latter approach with which we are recently being faced more and more. Does this mean that we are unable to continue asking just "how" things are, in order to grasp the ontology of the objects in question? Partially, yes! In fact, as formal sciences developed we discovered that it is often possible to describe the same class of phenomena or the same set of objects by means of more than one formal theory, each one of which reproduces the same observable values. Semantical analysis is then usually employed as a kind of a shortcut in order to eliminate all but one appropriate formal theory of the phenomena under consideration. However, such a pragmatic unification of science significantly distorts our naturalistic world-view, as we have argued on the example of quantum mechanics (QM).[1] For that reason we have proposed that all the existing rival theories be taken into account and appropriate semantics formulated in order to reconcile them. As the usual semantics cannot serve such a purpose (being constructed for quite another reason), our first aim is to formulate a new semantics—by now within the realm of QM—which would approve the undertaking and eventually enable us to get a better insight into the nature of natural sciences as well as into Nature itself.

In QM there are many rival theories (standard, statistical, stochastical, phase space, etc.) between which we cannot decide on an observational basis.[2] Every one of them offers a different ontology of the very same object under consideration. The first possible solution to this problem, as we have already mentioned, would be to choose between the rival semantics (i.e. between rival ontologies). Namely, according to the standard (Copenhagen) formulation, the quantum object does not have the proper trajectory, while according to the statistical interpretation it has.[3] Also, according to the stochastical theory a quantum object interacts with "some" subquantum structure, while according to the others it does not.[4] According to certain hidden-variables theories it obeys the causal law—according to the others it does not.[5] And so on. But how to choose the "proper"semantics when, at a present stage of scientific development, we have no empircal ground upon which to decide between any two of the theories? There is a pile of "objective" argumentation for every one of them (the standard, Copenhagen pile being especially huge). But there is also a standpoint, advocated most vigorously by Feyerabend, that we accept or reject a theory on the basis of a fundamentally "irrational" procedure. The very existence of other piles pours doubt on the objectivity of particular pile, while the other extreme, i.e. the underlying irrationality of our decisions, looks a bit too anarchistic at first sight. Still, it is the latter alternative which is historically approvable, although under the condition (as we have elaborated elsewhere[6]) of rendering "irrational" as "non-rational" with regard to the theory itself, i.e. the acceptance or rejection of a theory not based on the internal structure of the theory, but on external factors such as pragmatic, social, economical, traditional, ideological, etc. In any case it seems that science's main task to see "how" things are is being violated by this first approach to the non-uniqueness problem. (It is worth mentioning here that some years ago it seemed quantum logic could offer an empirical decision procedure, but it was soon recognized as just an abstract formulation of the standard interpretation of QM.)

Another possible solution to the non-uniqueness problem, the one we support, consists in adopting particular semantics to obtain the same hard formal core for all the theories and only afterwards to search for possible surplus structure of particular theories and find specific differences between them. At first sight such a programme seems to be hardly possible as the object logic of the standard formulation of QM is usually understood to be non-Boolean while the object logic which phase-space or stochastical formulatons obey is Boolean. However, such a result strongly depends on the attached semantics and in fact we can dispense with non-Boolean object logic within the standard formulation. The rest of the paper will deal with this problem.

Quantum logic (QL) is not, to repeat the claim given in the parenthesis above, a system we could read off from empircal situations and measurements results, despite frequent attempts to show it in this light,[7] for two main reasons: First, as we have already stressed, there are other axiomatizations, with different underlying logic, of the same empirical situation.[8] Second, for QL the very existence of incommensurable observables as well as the formal theory in which they are cast are essential, so that we cannot decide between classical and quantum logic solely on the empirical basis of YES–NO measurements (this is, in fact, the sense of Mackey's famous 7th axiom).[9]

QL is a system we read off from (the closed subspaces of) the Hilbert space formulation of QM. More precisely, QL is a system we get after ascribing a semantics (i.e. propositional values) to elements of the orthocomplemented quasimodular lattice (OQL), which is generated by the algebraic structure of subspaces of Hilbert space. As Gleason, Kamber, Kochen and Specker have shown, we cannot ascribe the truth-value semantics to the lattice. In a substitution we are used to ascribing intuitionistic, i.e. operationalistic semantics, adhering to measurement situations.[10] However, a rather curious antagonism appears in doing so. Namely, if we limit ourselves exclusively to empirical measurement situations, we cannot include the superposition of states in the structure of QM, except through the material propositons. This means that in this case only incommensurability (complementarity) is responsible for breaking the distributive law (non-Boolean feature of QL) and even then only under the assumption that the lattice is read off the Hilbert space structure. On the other hand, to the lattice whose structure includes superpositon we cannot attach the proper semantics, i.e. we can no longer speak about proper logic. Besides, it has been proved that the principle of superposition and the principle of complementarity cannot be reduced one to the other, and that they break the distributivity law independently.[11] In conclusion, if we were able to split the semantics of the lattice into two independent parts, i.e. to differentiate between that part of the Hilbert space description which corresponds to internal features (superposition, atomic structure, etc.) and that which corresponds to external ones (measurement), we would also be able to resolve the exposed antagonism. In a word, we should be able to show that the wave functions which correspond to superposition and those which correspond to measurement are not the same—and that the former are interconnected (superposed) while the latter are not.Such a proof would seem to be in sharp contradiction to the claimed measurement interconnectedness and action-at-a-distance underlying the mutually exclusive measurements; claims which we have become used to inferring from the so-called non-locality property within the Bell type experiments. However such an inference is unjustified and stems from misconceived counterfactuals.

To prove the last statement, we shall consider Bell's QM system consisting of two electrons of total spin 0, separated in space, and moving in opposite directions. If one observer detects the spin projection of the first electron, with the help of Stern-Gerlach magnets, along directions $\vec{a}$ or $\vec{a}'$, and another detects the spin projection of the second electron along directions $\vec{b}$ or $\vec{b}'$, the non-locality property of QM may appear. What does this mean? If the first observer orients his magnets (detector) along $\vec{A}$ and the second along $\vec{B}$, they can measure, in a single, i-th run, the following quantity: $c_i(A,B) = n_i(+/A)\, n_i(+/B) + n_i(-/A)\, n_i(-/B) - n_i(+/A)\, n_i(-/B) - n_i(-/A)\, n_i(+/B)$, where $n_i(-/B)$,e.g., corresponds to detecting (= 1) or not detecting (= 0) the spin of the second electron as being oriented down when measured along $\vec{B}$, by the second observer. Correlation is then, in the long run, determined by (the sum goes from $i = 1$ to $i = N$):

$$C(A,B) = \frac{1}{N}\Sigma c_i(A,B) \approx P(+,+/A,B)+P(-,-/A,B) - P(+,-/A,B) - P(-,+/A,B) =$$
$$= \frac{1}{2}\cos^2[(A,B)/2] + \frac{1}{2}\cos^2[(A,B)/2] - \frac{1}{2}\sin^2[(A,B)/2] - \frac{1}{2}\sin^2[(A,B)/2] =$$
$$= \cos(A,B),$$

where $P(+,-/A,B)$, e.g., is the probability with which the correlated electron could be found to have spin projections oriented up and down when measured along $\vec{A}$ and $\vec{B}$, respectively. In the i-th run, for four different orientations of the detectors, one real and three others counterfactually assumed, the following expression may be formed:

$$s_i = c_i(a,b) - c_i(a,b') + c_i(a',b) + c_i(a',b').$$

In the long run it gives:

$$S = \frac{1}{N}\Sigma s_i = C(a,b) - C(a,b') + C(a',b) + C(a',b').$$

The non-locality QM feature now means that in the long run $|S|$ can acquire values which are greater than 2 in contrast with $|s|$ which may, it is commonly held, acquire a value not greater than 2 under the assumption that a measurement made by the first observer along $\vec{A}$ cannot be influenced by the orientation ($\vec{B}$) of the detector of the second observer. From this contrast it is often concluded that the sub-systems are interconnected, that there is some action-at-a-distance—presumably superluminal, that the EPR objection is invalid and that the wave function which describes that intercorrelated sub-systems in fact describes an internal, objectively existing interconnectedness.[12] However, this conclusion is unjustified and stems from a misconceived analysis.[13] To prove this, let us look at the correlations for the situation in which the first detector is held fixed and oriented along $\vec{a}$ and the second oriented along $\vec{b}$ and $\vec{b}'$, respectively: $C(a,b)$ and $C(a,b')$. The following connections between frequencies and probabilities (for four particular terms of $C(a,b)$, placed on the left hand side, and of $C(a,b')$, placed on the right hand side) are to be obtained:

$$\frac{1}{N}\Sigma n_i(+/a)\, n_i(+/b) \approx \frac{1}{2}\cos^2[(a,b)/2]; \quad (1) \quad \frac{1}{N}\Sigma n_i(+/a)\, n_i(+/b') \approx \frac{1}{2}\cos^2[(a,b')/2];$$
$$\frac{1}{N}\Sigma n_i(-/a)\, n_i(-/b) \approx \frac{1}{2}\cos^2[(a,b)/2]; \quad (2) \quad \frac{1}{N}\Sigma n_i(-/a)\, n_i(-/b') \approx \frac{1}{2}\cos^2[(a,b')/2];$$
$$\frac{1}{N}\Sigma n_i(+/a)\, n_i(-/b) \approx \frac{1}{2}\sin^2[(a,b)/2]; \quad (3) \quad \frac{1}{N}\Sigma n_i(+/a)\, n_i(-/b') \approx \frac{1}{2}\sin^2[(a,b')/2];$$
$$\frac{1}{N}\Sigma n_i(-/a)\, n_i(+/b) \approx \frac{1}{2}\sin^2[(a,b)/2]; \quad (4) \quad \frac{1}{N}\Sigma n_i(-/a)\, n_i(+/b') \approx \frac{1}{2}\sin^2[(a,b')/2];$$

One then usually argues counterfactually: "If the second observer suddenly switches the detector from $\vec{b}$ to $\vec{b}'$ then the result which the first observer obtains does not coincide with the one he would obtain if the second detector were not switched. For, otherwise they could not get the appropriate frequencies (quoted above)". That such a conclusion is misplaced we can easily recognize after adding (1) + (3) and (2) + (4) on each side:

$$\Sigma n_i(+/a)\, n_i(+/b) + \Sigma n_i(+/a)\, n_i(-/b) \approx \frac{N}{2}; \quad \Sigma n_i(+/a)\, n_i(+/b') + \Sigma n_i(+/a)\, n_i(-/b') \approx \frac{N}{2}$$
$$\Sigma n_i(-/a)\, n_i(-/b) + \Sigma n_i(-/a)\, n_i(+/b) \approx \frac{N}{2}; \quad \Sigma n_i(-/a)\, n_i(-/b') + \Sigma n_i(-/a)\, n_i(+/b') \approx \frac{N}{2}$$

As n_i's in different sums in each equation cannot coincide, and as $n_i\,(./B)$'s are equal to 1 or 0 simultaneosly with corresponding $n_i\,(./a)$'s in particular equations, we get:

$$\Sigma n_i\,(+/a) \approx \frac{N}{2}, \vec{B} = \vec{b}; \quad \Sigma n_i\,(-/a) \approx \frac{N}{2}, \vec{B} = \vec{b}\ ;$$

$$\Sigma n_i\,(+/a) \approx \frac{N}{2}, \vec{B} = \vec{b}'; \quad \Sigma n_i\,(-/a) \approx \frac{N}{2}, \vec{B} = \vec{b}'.$$

Thus particular detections the first observer gets does not depend upon the orientations of the second observer's detector–the observers just *arrange* and *classify* them depending on orientations (i.e. upon registered data–which is usually hidden behind an automatic "coincidence detection"[14]) of the second observer's detector. Namely, $N/2$ detections of $n_i\,(+/a)$ and $N/2$ detections of $n_i\,(-/a)$ are just being distributed in equations (1) and (3), and in (2) and (4), respectively, depending on values of $n_i\,(\,.\,/b)$'s (when we distribute them in the left hand side equations) and $n_i\,(\,.\,/b')$'s (when we distribute them in the right hand side equations); they themselves need not change. It is therefore evident (we can make an analogous elaboration for the first detector being oriented along $\vec{a}'$) that no action-at-a-distance is responsible for the quantum nonlocality result, but solely the methodological necessity to combine (in correlations, i.e. probabilities) for example $n_i\,(+/a)$'s somtimes with $n_i\,(+/B)$'s and sometimes with $n_i\,(-/B)$'s, depending on the value of $\vec{B}$–in order to get the correct probabilities: the quantum ones:[16]

$$\frac{1}{2}\cos^2[(A,B)/2] \text{ and } \frac{1}{2}\sin^2[(A,B)/2].$$

(If we were to apply the corresponding classical probabilities: $1/2 - |(A,B)|/2\pi$ and $|(A,B)|/2\pi$, we would obtain the locality result: $|S| \leq 2$.) Whenever a quantum particle transmits information (for a classical system we are transmitters of information) through a quantum binary channel we are bound (for a classical system we are free to formulate statistics e.g. on a geometrical basis) to connect the resulting frequencies with the wave function probabilities. Thus, on the one hand, we recognize the wave function as being a result of transmitting the information through the measurement channel, and the measurement itself as picking up the value of the particular observable in question, while on the other hand, the wave function does independently describe objective internal properties such as interference, atomic structure, etc. In effect, we can split the QM structure into two parts: internal description given by "internal" wave function, and external descripton given by YES-NO binary "external" (measured) wave function. They are connected through the projection postulate, but are nevertheless fundamentally different in the operational sense, as shown below. The internal description gives us a structure of internal possibilities. The external description stems from the interaction of a system with a measuring apparatus when obtaining the information on a particular observable.

The underlying logic of the above descriptions can be determined in the following way. Through experiment, the real valued experimental function f, from all states into $[0,1]$, is uniquely determined. We can operate on such experimental functions as on real functions: If f and g are elements of the set of all the functions, L, then $f+g$ denote the function $f(x)+g(x)$, where x is from the set of all states; $f \leq g$ means $f(x) \leq g(x)$, etc. Such functions are obviously orthogonal and we can prove that the set L, which is merely what is commonly called a logic of propositions about the system, is an orthocomplemented partially ordered (OPO) set with respect to the natural order of real functions (complementation: $f' = 1-f$). If the observables are commensurable, the L (logic) is Boolean. But what happens if the observables are complementary? In the literature this usually serves as ground for the inference of a non-Boolean algebra. But the expressions such as $a \wedge b$, e.g., are completely devoid of any specific semantics. We can state $a \wedge b = 0$, with the consequence that a is not orthogonal to b, in the sense of a definition of the complementarity of a and b. In this case, however, we cannot speak about the proper identification of propositions with empirically confirmed relationships among observables. We can only speak about meta-statements, pronounced long ago by Bohr, which express some kind of superselection rules among classical measurements; about Boolean structures which are just glued together into the OPO set of orthogonal projections on Hilbert space. Thus the only way to take superposition into "logical account" is to interpret it as a material proposition.[15] I.e. we cannot logically interpret all the formulas of PO set L, as identified with those of the OQL of Hilbert space, simply as they stand, but must treat some of them as propositions for themselves. In other words, the interpretation the OQL (otherwise simply called QL) can also be split into two parts, in the same way as with the basic structure above. The first, internal part, in which the distributivity law is broken by superposition of states, is not a proper logic at all, but a set of instructive meta-rules (SIM) for designing the preparatory conditions and wave functions and operators. The second part is not a proper logic either, but a meta-logic (ML) which designs the meaning and usage of particular terms in respective object logics (!), which are themselves Boolean. For lack of space, we will just illustrate the afore-said by the two-slit experiment where a particle passes through a screen with two slits, A and B, and reaches the detecting screen C. The lattice structure of Hilbert space would first, understood as SIM, tell us that $(A \vee B) \wedge C$ means "both slits are open and the hitting of C is being detected; apply appropriate wave function accordingly (aawfa)", while $(A \wedge C) \vee (B \wedge C)$ means "(just A is open and detection is being made on C) or (just B is open and detection on C); aawfa". Thus the broken distributive law: $(A \vee B) \wedge C \neq (A \wedge C) \vee (B \wedge C)$ simply means nonequivalence of preparatory conditions (experimental arrangements). The lattice structure understood as ML would put a meta-demand that A,B and C in $(A \wedge C) \vee (B \wedge C)$ are to be interpreted as propositions, while in $(A \vee B) \wedge C$, $(A \vee B)$ and C are to be interpreted as propositions, i.e. $A \vee B$ is to be treated as an elementary proposition: $e\ (A \vee B)$. The broken "distributive" law would in this case mean that we are dealing with two conceptually incomparable measurable situations, each of which, however, rests on a Boolean logical structure, and would be represented by: $e\ (A \vee B) \wedge C \neq (A \vee C) \vee (B \wedge C)$. E.g., probability of the situation on the right is: $P\ ((A \wedge C) \vee (B \wedge C)) = \langle\psi \mid P_A P_C P_A \mid \psi\rangle + \langle\psi \mid P_B P_C P_B \mid \psi\rangle$, where P_i are respective projectors. As P_C commutes with P_A and P_B, and $P_A P_B = 0$, the equation is also equal to: $\langle\psi \mid P_A + P_B) P_C (P_A + P_B) \mid \psi\rangle = P\ (\ (A \vee B) \wedge C)$. The probability of the situation on the left (interference), $P\ (e(A \vee B) \wedge C)$, deserves yet another comment: we might be tempted to use the formally equivalent expression from SIM:

$$\langle\psi \mid P_{A \vee B} P_C P_{A \vee B} \mid \psi\rangle = \langle\psi \mid P_A + P_B) P_C (P_A + P_B) \mid \psi\rangle =$$
$$= \langle\psi \mid P_A P_C P_A \mid \psi\rangle + \langle\psi \mid P_B P_C P_B \mid \psi\rangle + \langle\psi \mid P_A P_C P_B + P_B P_C P_A) \mid \psi\rangle$$

It *is* the proper QM expression, but as P_C does not commute either with P_A or with P_B, $P_C P_A$ and $P_C P_B$ are no longer observables (nor projectors) and, as such, are devoid of measurable physical meaning, i.e. they cannot be *separately* interpreted as (measurable) proposition in any logic. This also clarifies why we are not able to speak about non-classical probabilities: The rules of a probability theory being valid only when there is one invariable complex of conditions under which random tests are carried out, when we replace the invalid inequality: $P\ ((A \vee B) \wedge C) \neq P\ ((A \wedge C) \vee (B \wedge C))$, often cited in literature, with the adequate one: $P\ (e(A \vee B) \wedge C) \neq P\ ((A \wedge C) \vee (B \wedge C))$, we can easily see that there is no one invariable complex of conditions underlying both probabilities. This last fact, together with the already disproved action-at-a-distance underlying the Bell measurements, show that it is not the measurement problem, but internal "superposition" of individual objects that cannot be "explained" by any logic or probability structure. This justifies investigation of rival theories and their further elaborations, from yet another standpoint.

ENDNOTES

1 Pavičić, M., “A Demarcation in the Ontology of the Naturalistic World-View”, in: W. Leinfellner, E. Kraemer und J. Schank (eds.), *Sprache und Ontologie* (Wien 1982), pp. 354–7; M. Pavičić, “How Many Truths Are We Responsible for?”, a paper read in the seminar “Science and Technology–Future–Responsibility” (Kirchberg am Wechsel 1982) (in print).

2 See references cited in Pavičić (1982a) and Pavičić (1982b).

3 Mayants, L.S., Yourgrau, W. and van der Merwe, A.J., “Some Methodological Problems in Quantum Physics”, *Annalen der Physik*, Vol. 33 (1976), pp. 21–35.

4 Aron, J.C., “Stochastic Foundation for Microphysics”, *Found. Phys.* Vol. 11 (1981), 699–719.

5 Bohm, D. and Bub, J., “On Hidden Variables”, *Rev. Mod. Phys.* Vol. 40 (1968), pp. 235–6.

6 Pavičić (1982a).

7 See Bohm & Bub (1968) and H. Putnam, “QM and the Observer”, *Erkenntnis* Vol. 16 (1981) 193.

8 Stochastical, phase space and fuzzy phase space are the most developed ones.

9 Bugajski , S. and Lahti, P.J., *Int. J. Theor. Phys.* Vol. 19 (1980), pp. 499–514.

10 Hooker, C.A. (ed.), *Physical Theory as Logico-Operational Structure* (Dordrecht 1979).

11 Lahti, P.J., *Int. J. Theor. Phys.* Vol. 19 (1980), pp. 789–842; Bugajski & Lahti (1980).

12 Aspect, A. *et al., Phys. Rev. Lett.* Vol. 49 (1982), pp. 91–4 (action-at-a-distance); A. Garuccio *et al., Lett. Nuovo Cim.* Vol. 32 (1981), pp. 451–6 (a superluminal one).

13 Pavičić, (1983) (in print).

14 Aspect, A. *et al.* (1982).

15 Stachow, E.-W., “An Operational Approach to Quantum Probability”, in: Hooker (1979), p. 285.

16 Namely, the fourth possible situation: $\{\vec{a}', \vec{b}'\}$, represents a new experimental arrangement and does not endanger the result. I.e., when we measure $c_i(a, b,) = xy$, and assume $c_i(a, b') = xz$ and $c_i(a', b) = wy$, then according to no locality the product, wz, of data from the corresponding experimental arrangements, is necessarily equal to a possible experimental outcome, $c_i(a', b')$, of a new experimental arrangement.

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