A New Class of 4-Dim Kochen-Specker Sets

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Kochen-Specker theorem

The Kochen-Specker theorem amounts to the following claim: In $\mathcal{H}^n$, $n \geq 3$, it is impossible to assign 1s and 0s to all vectors in such a way that

1. No two orthogonal vectors are both assigned 1;
2. In any subset of n mutually orthogonal vectors, not all of the vectors are assigned 0.

KS vectors in each KS set form subsets of n mutually orthogonal vectors.

We arrive at one subset from another by a series of rotation in 2-dim planes around $(n-2)$-dim.

Orthogonal Spins

We have to measure spins in 3, 4, 5, . . . dimensions.

Of course in a Hilbert space.

Vectors are orthogonal $\Rightarrow$ nonlinear equations

$$\mathbf{a}_B \cdot \mathbf{a}_C = a_{B1}a_{C1} + a_{B2}a_{C2} + a_{B3}a_{C3} + a_{B4}a_{C4} = 0,$$

$$\mathbf{a}_B \cdot \mathbf{a}_D = a_{B1}a_{D1} + a_{B2}a_{D2} + a_{B3}a_{D3} + a_{B4}a_{D4} = 0,$$

$$\mathbf{a}_B \cdot \mathbf{a}_E = a_{B1}a_{E1} + a_{B2}a_{E2} + a_{B3}a_{E3} + a_{B4}a_{E4} = 0,$$

$$\mathbf{a}_C \cdot \mathbf{a}_D = a_{C1}a_{D1} + a_{C2}a_{D2} + a_{C3}a_{D3} + a_{C4}a_{D4} = 0,$$

$$\mathbf{a}_C \cdot \mathbf{a}_E = a_{C1}a_{E1} + a_{C2}a_{E2} + a_{C3}a_{E3} + a_{C4}a_{E4} = 0,$$

$$\mathbf{a}_D \cdot \mathbf{a}_E = a_{D1}a_{E1} + a_{D2}a_{E2} + a_{D3}a_{E3} + a_{D4}a_{E4} = 0.$$

Mission Impossible

To solve these equations for all possible combinations for at least 18 vectors (no solutions below 18) we would need a million ages of the universe on all today's processors on the Globe working in parallel

Use hyper-graphs instead of equa-tions and vectors.

Exponential $\Rightarrow$ *Polynomial*

We first “translate” nonlinear equations into linear hypergraphs, diagrams, MMP diagrams.

Next, we impose conditions on generation of hypergraphs. Generation proves to be statistically polynomially complex (SPC).

We filter the obtained hypergraphs by additions conditions. The procedure is also SPC.

In the end we translate a rather “small” number (millions) of hypergraphs back into equations and solve them by means of interval analysis method. Its programs are SPC as well.

Algorithm

Vectors are vertices (points) and orthogonalities between them are edges (lines connecting vertices).

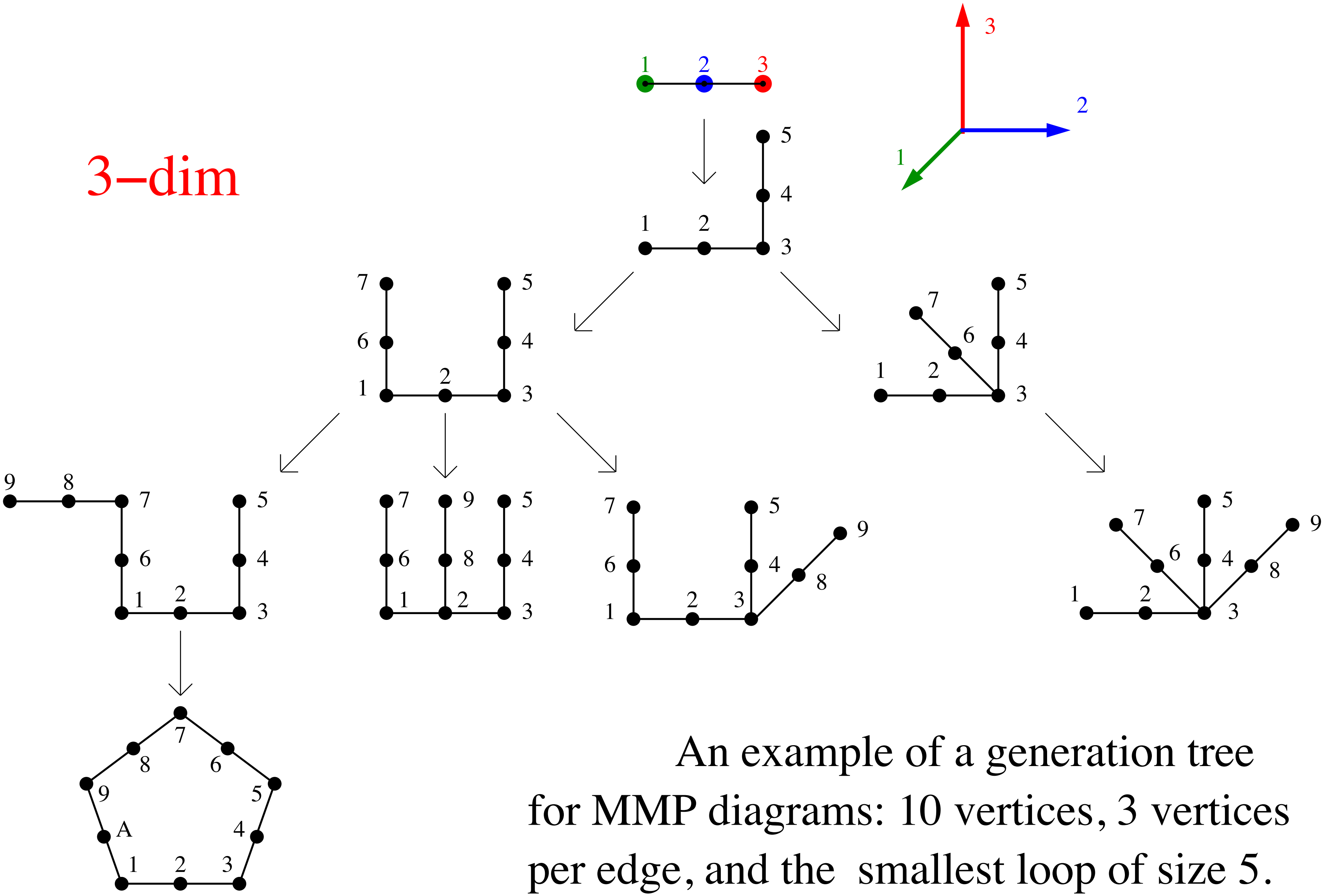
Thus we obtain MMP diagrams which are defined as follows:

1. Every vertex belongs to at least one edge;
2. Every edge contains at least 3 vertices;
3. Edges that intersect each other in $n-2$ vertices contain at least n vertices;

We denote vertices of MMP diagrams by `1,2,..,A,B,..a,b,...` There is no upper limit for the number of vertices and/or edges in our algorithms and/or programs.

Generation of MMP diagrams

3–dim



An example of a generation tree for MMP diagrams: 10 vertices, 3 vertices per edge, and the smallest loop of size 5.

Solutions!!??

\	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*total*
18	1																1
19		1															1
20		1	4+1	1													7
21			2	11	4	1											18
22			1	9	36	23	12	3	1								85
23				2	19	76	79	58	27	11	3	1					276
24				1	6	39	137	187	188	136	83	40+1	18	6	2	1	845
total	1	2	8	24	65	139	228	248	216	147	86	42	18	6	2	1	1233

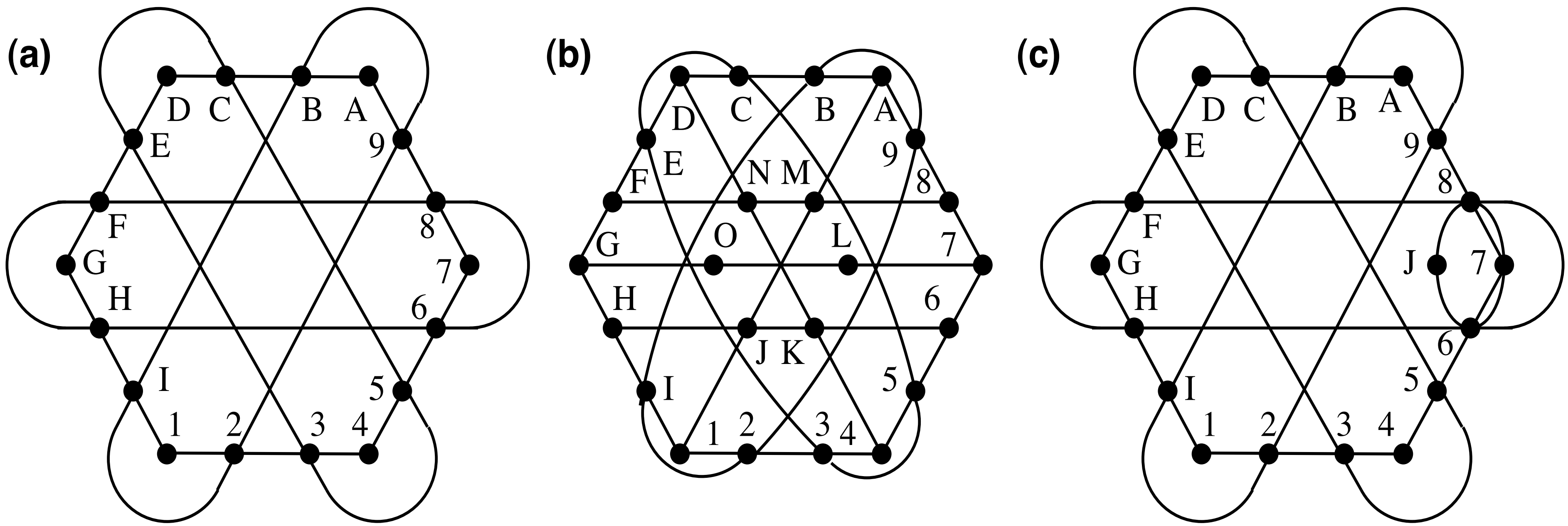
Boxed in red are the solutions previously found by humans.

It would take over 150 years to generate all the solutions on a single PC

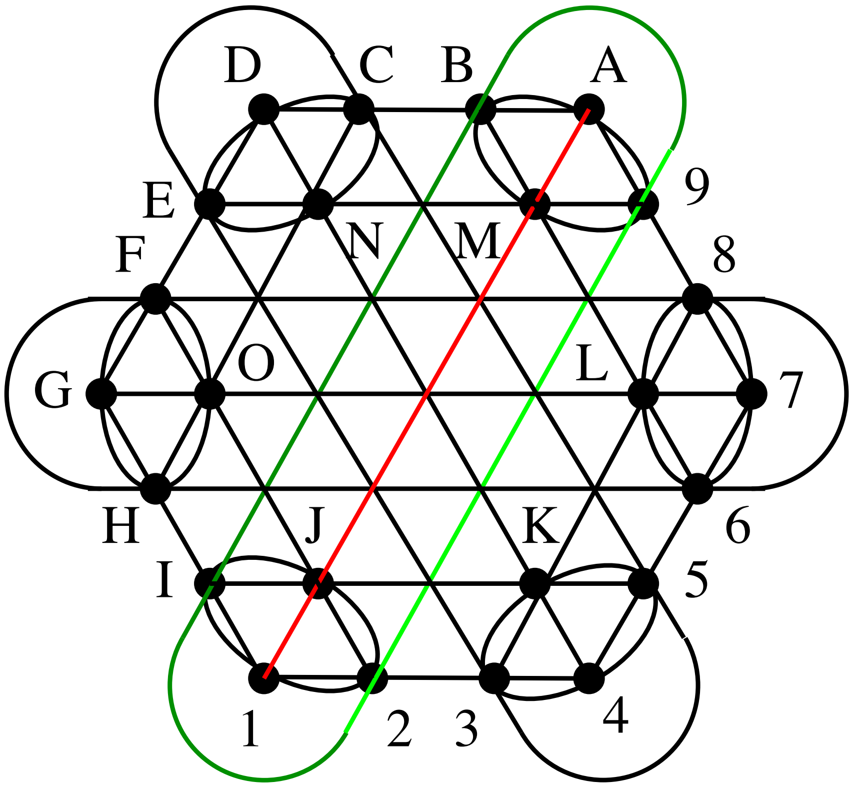
Let us waste some CPU-weeks on a 500 CPU cluster

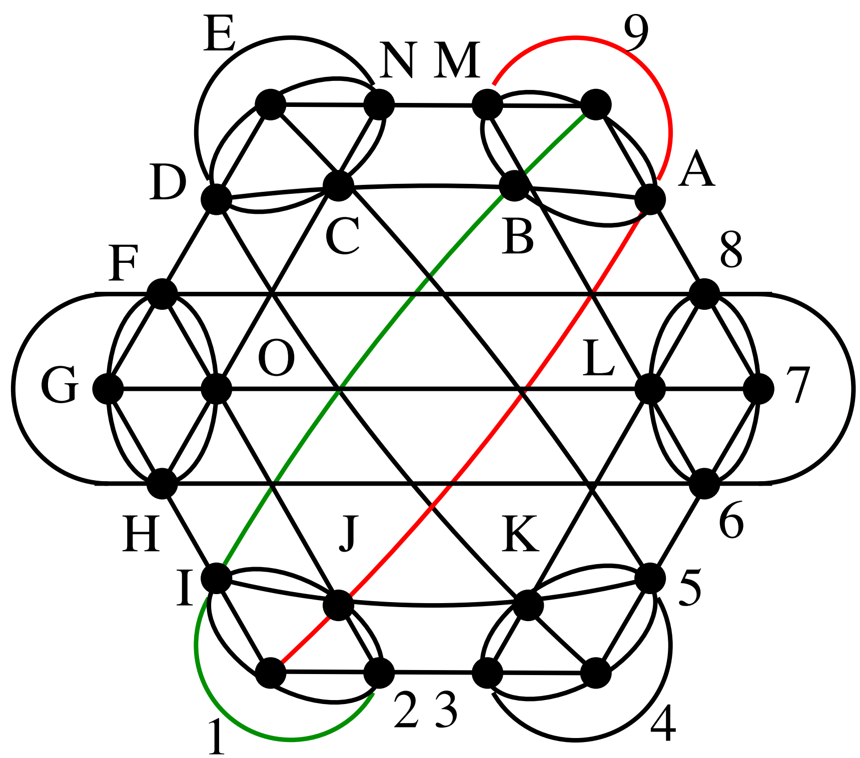
What have we obtained?

How do our solutions look like?



4-dim 24-24 MMP diagram





4-dim 24-24 MMP diagram

Let us peel off our 24-24 diagram!

It takes 20 min on a single PC to get all solutions.

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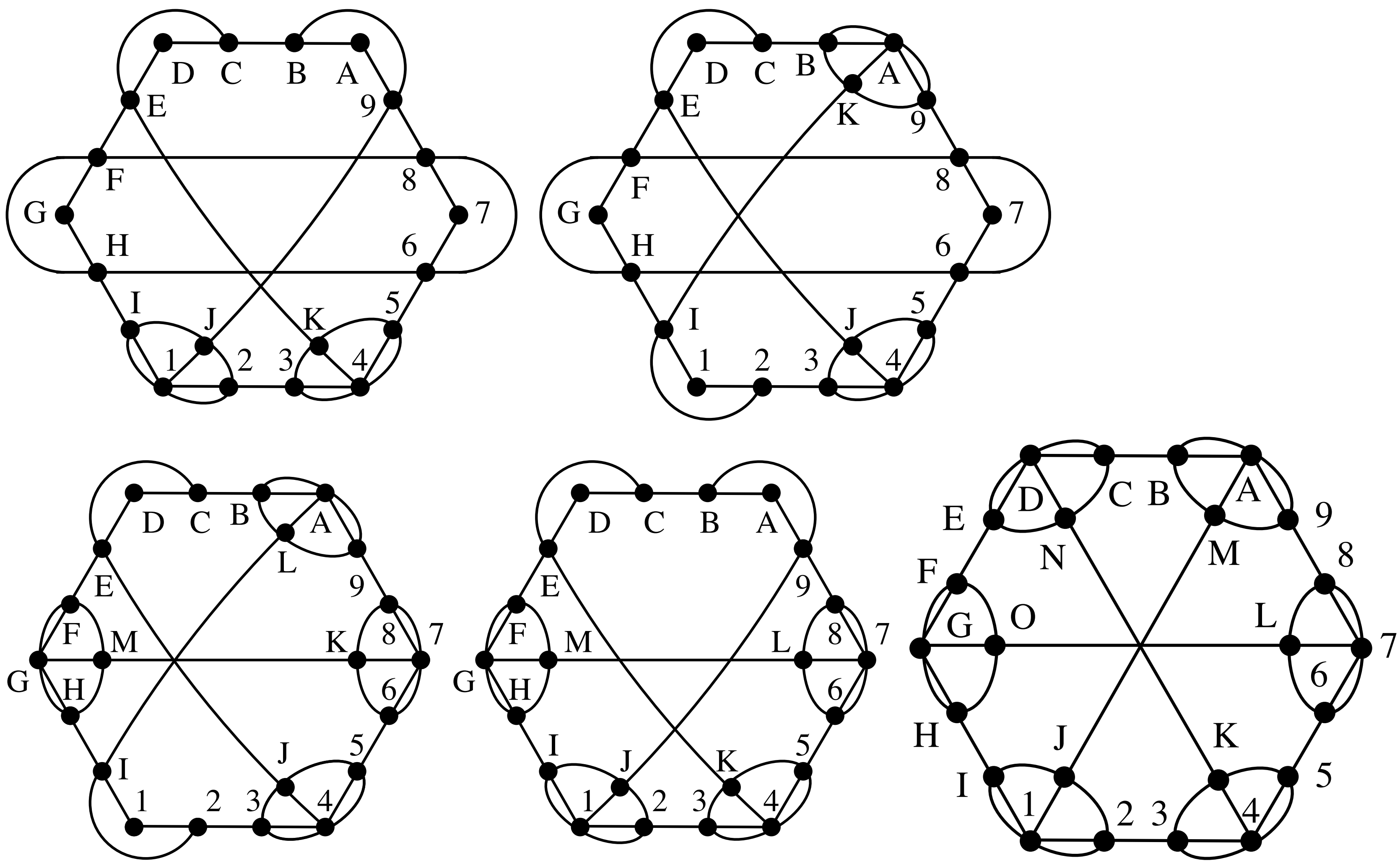
Criticals: 20-11, 22-13, 24-15



Parity Proofs for Critical KS Sets

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(a) and (b) clash, so no predetermined 0,1 values can be ascribed to the vertices.

Symmetries and Enlargements

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  - Physically relevant KS sets are minimal ones: the so-called *critical sets*.
- There are only 6 critical sets in the 24 class.
  - No extension of MMP diagrams with 24 vectors and 24 tetrads has a solution.

Aravind's KS 60-75 set

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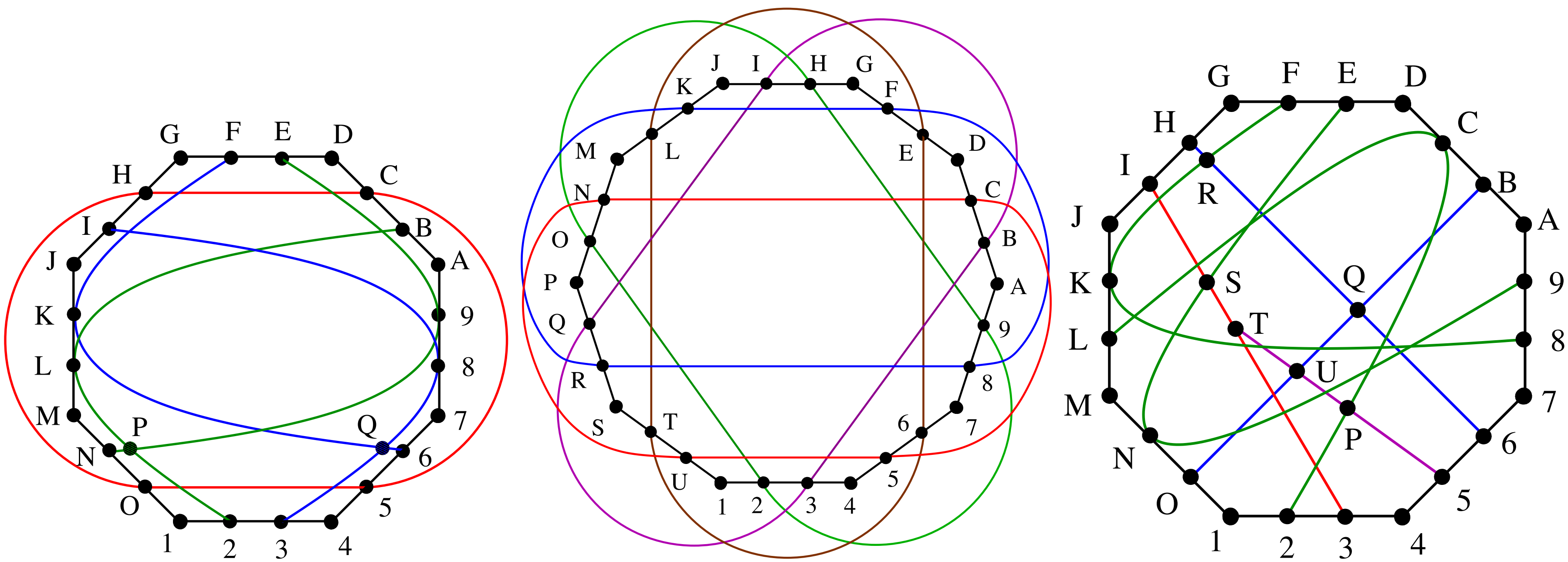
26-13 and two 30-15 critical sets:



MMP notation

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26-13
1234,4567,789A,ABCD,DEFG,GHIJ,JKLM,MNO1,5CHO,3Q8I,6QKF,NP9E,2PLB

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1234,4567,789A,ABCD,DEFG,GHIJ,JKLM,MNO1,5CHO,3Q8I,6QKF,NP9E,2PLB

30-15

1234,4567,789A,ABCD,DEFG,GHIJ,JKLM,MNOP,PQRS,STU1,6ELT,8FKR,C5UN,O29H,B3QI

MMP notation

26-13

1234,4567,789A,ABCD,DEFG,GHIJ,JKLM,MNO1,5CHO,3Q8I,6QKF,NP9E,2PLB

30-15

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60-75

1234,1cKT,1Qtg,1Njo,1yYE,2Mmn,2vZD,2Pri,2bIV,3HWe,3kqO,3XGx,3shS,4Fwa,
4UdJ,4fRu,4pLI,5678,5pSK,5XiN,5buE,5Wwm,9ABC,9fxK,9sVN,9PIE,9qdZ,AUOt,
AHiy,Abao,AGRm,BFSj,BXnc,BvJg,BWLr,CpeY,CkDQ,CMuT,ChwI,6Fet,6kVy,6PJo,
6hLZ,7Uxj,7sDc,7Mag,7qRI,8fOY,8HnQ,8vlT,8Gdr,FGDE,FqiT,UhnE,UWVT,fhig,
fWDo,pGVg,pqno,HIJK,HZuj,kraK,kmlj,XIlt,XZaY,srut,smJY,MLON,MdSy,vReN,
vwxy,PRSQ,PwOc,bLxQ,bdec.

32-17, two 33-17, and three 34-17

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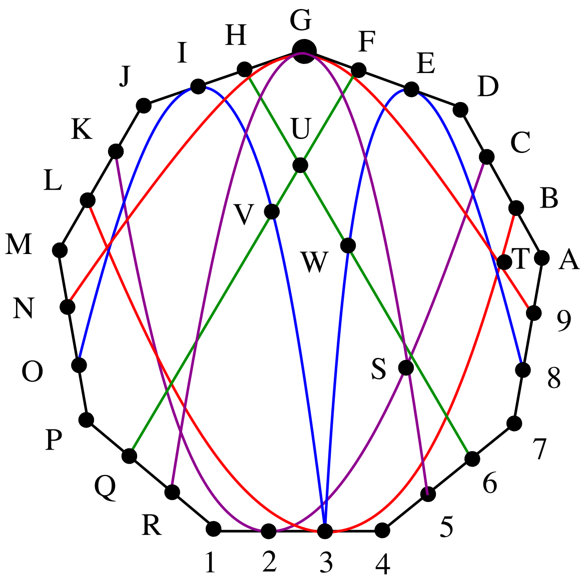

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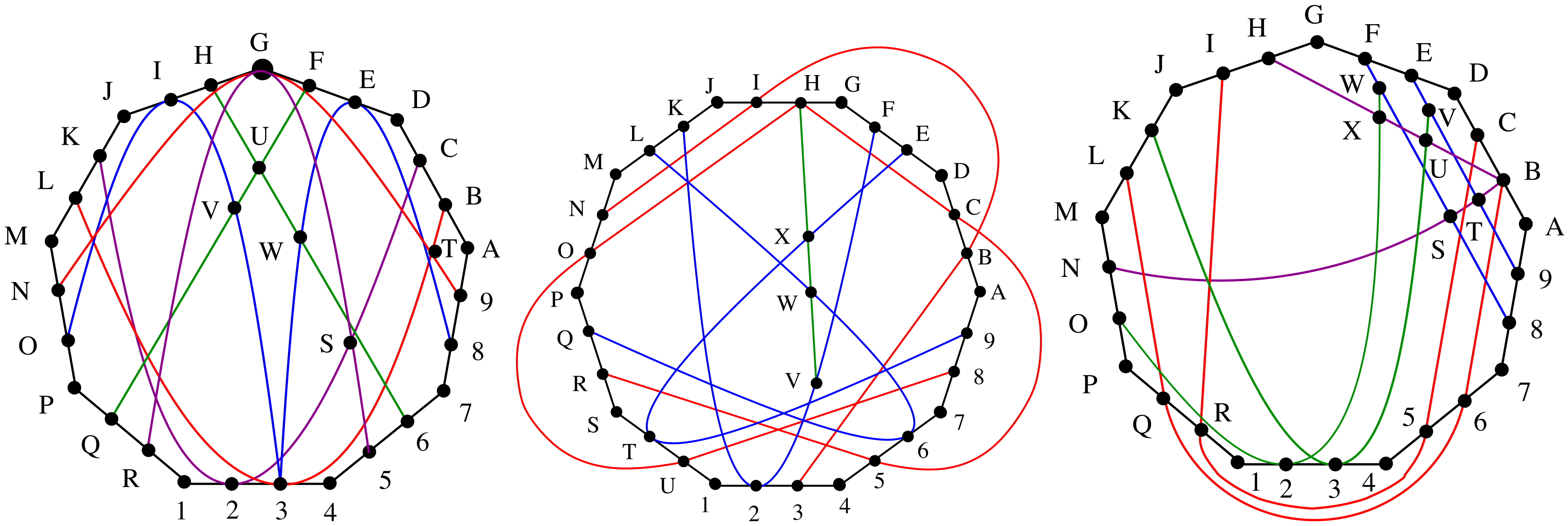

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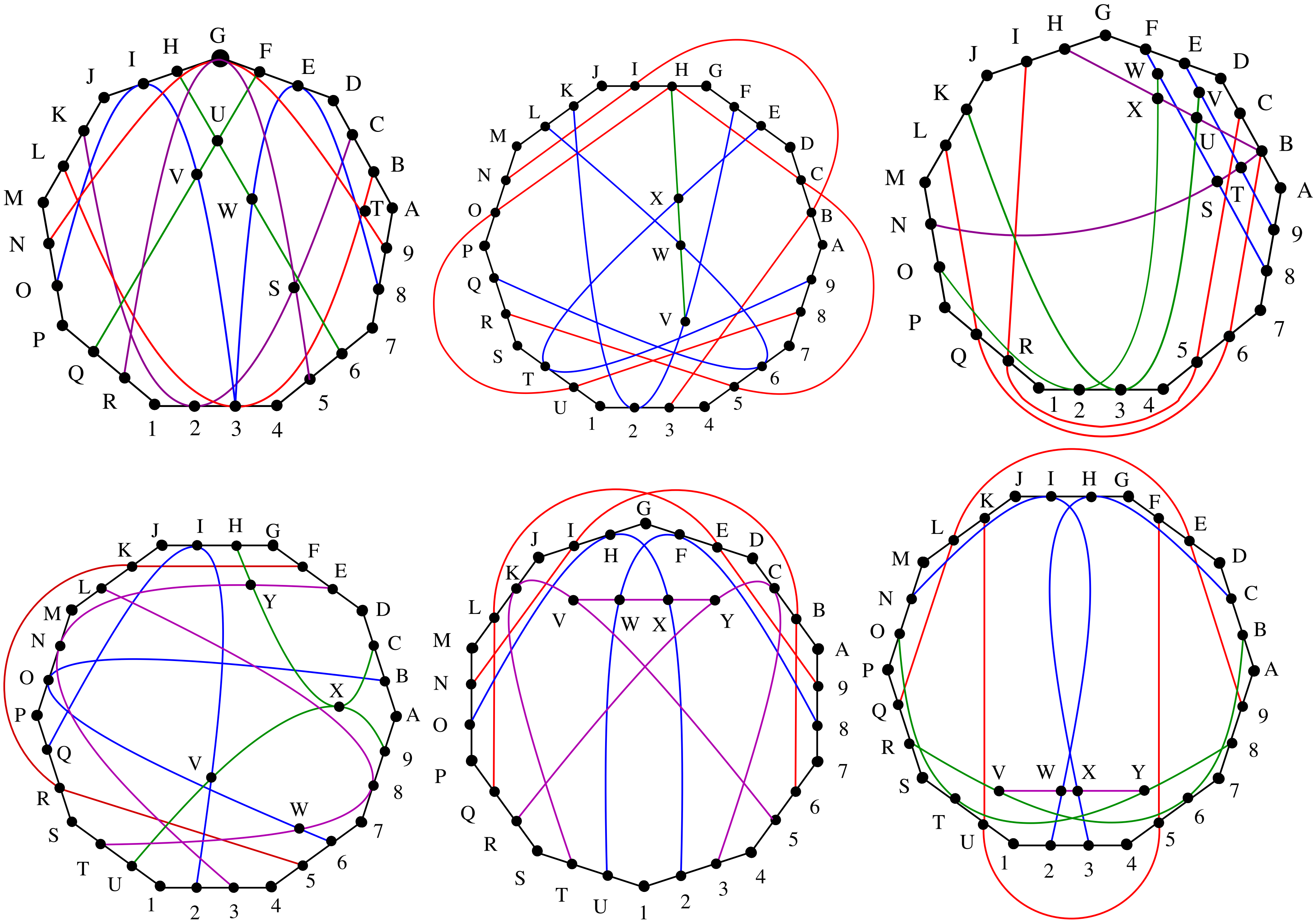

critical KS sets obtained so far

13	26
15	30
17	32,33,34
19	36,37,38
21	40,41,42
23	40,41,42,43,44,45,46
25	40,42,43,44,45,46,47,48,49,50
27	44,45,46,47,48,49,50,51,52,53,54
29	45,46,47,48,49,50,51,52,53,54,55
31	48,49,50,51,52,53,54,55,56,57,58
33	48,49,50,51,52,53,54,55,56,58
35	50,51,52,53,54,55,56,57,58
37	50,51,52,53,54,55,56,57,58,59,60
39	53,54,55,56,57,58,59,60
41	53,54,55,56,57,58,59,60
43	54,55,56,57,58,59,60
45	54,55,56,57,58,59,60
47	56,57,58,59,60
49	56,57,58,59,60
51	58,59,60
53	59,60
55	60

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Hexagon KS sets from up to 24 vector sets are the only ones that exist. Why?

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24 KS class offers only parity proofs for critical sets. 60 KS class has hundreds of them with and without parity.

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Hexagon KS sets from up to 24 vector sets are the only ones that exist. Why?

Is 60 KS class the only bigger class?