

# Why is the symmetry energy so uncertain at high densities?

Bao-An Li & collaborators:



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1. A quick overview of many-body theoretical predictions  
(frustrating but also stimulating!)
2. What are the controlling parameters?  
(very uncertain but they are the most important underlying physics!)
  - a) Effects of the tensor force and short-range correlations
  - b) Effects of the 3-body force
3. Is the super-soft symmetry energy physical based on known physics?
4. How to experimentally probe the high density symmetry energy?

# What is the Equation of State of neutron-rich nuclear matter?

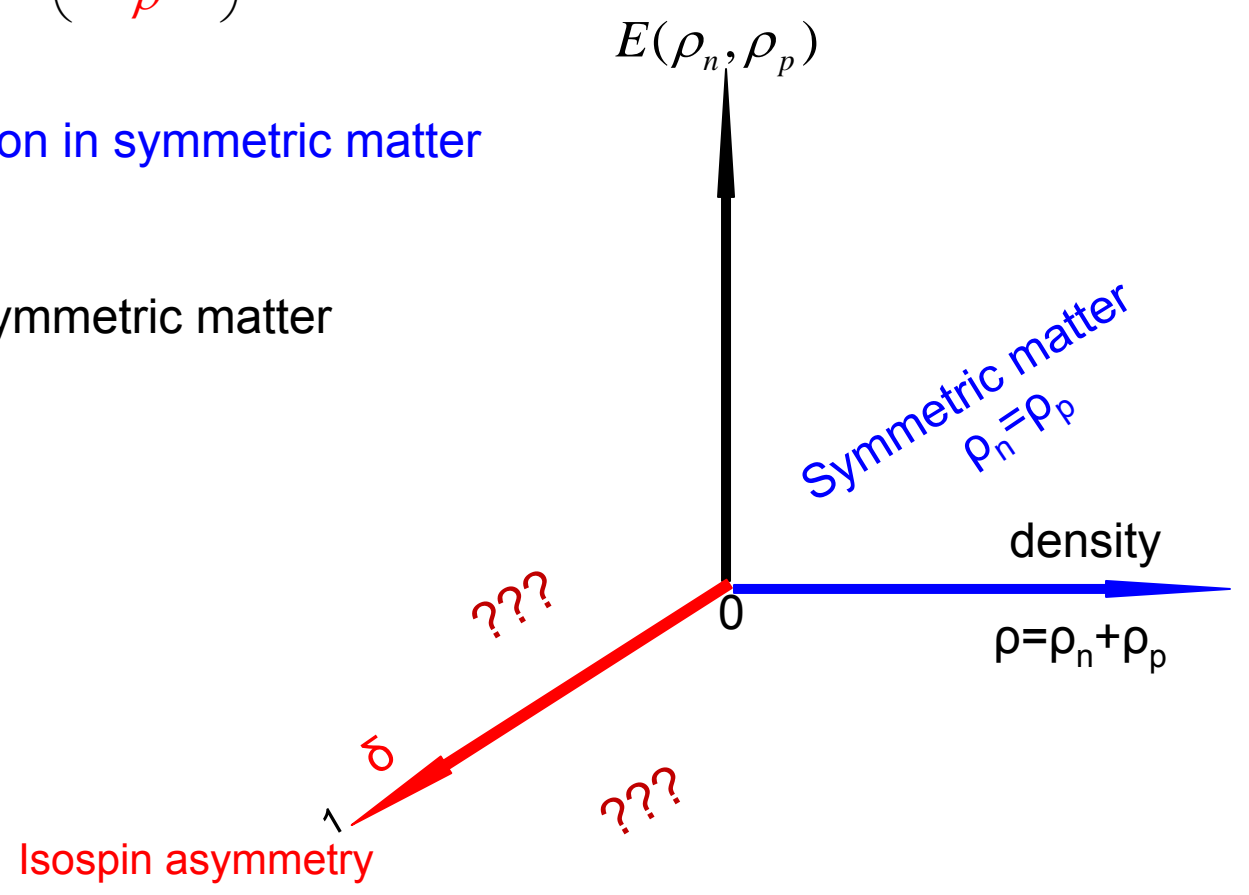
$$E_{sym}(\rho) = \frac{1}{2} \frac{\partial^2 E}{\partial \delta^2} \Big|_{\delta=0} \sim E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

symmetry energy      Isospin asymmetry  $\delta$

$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{sym}(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 + o(\delta^4)$$

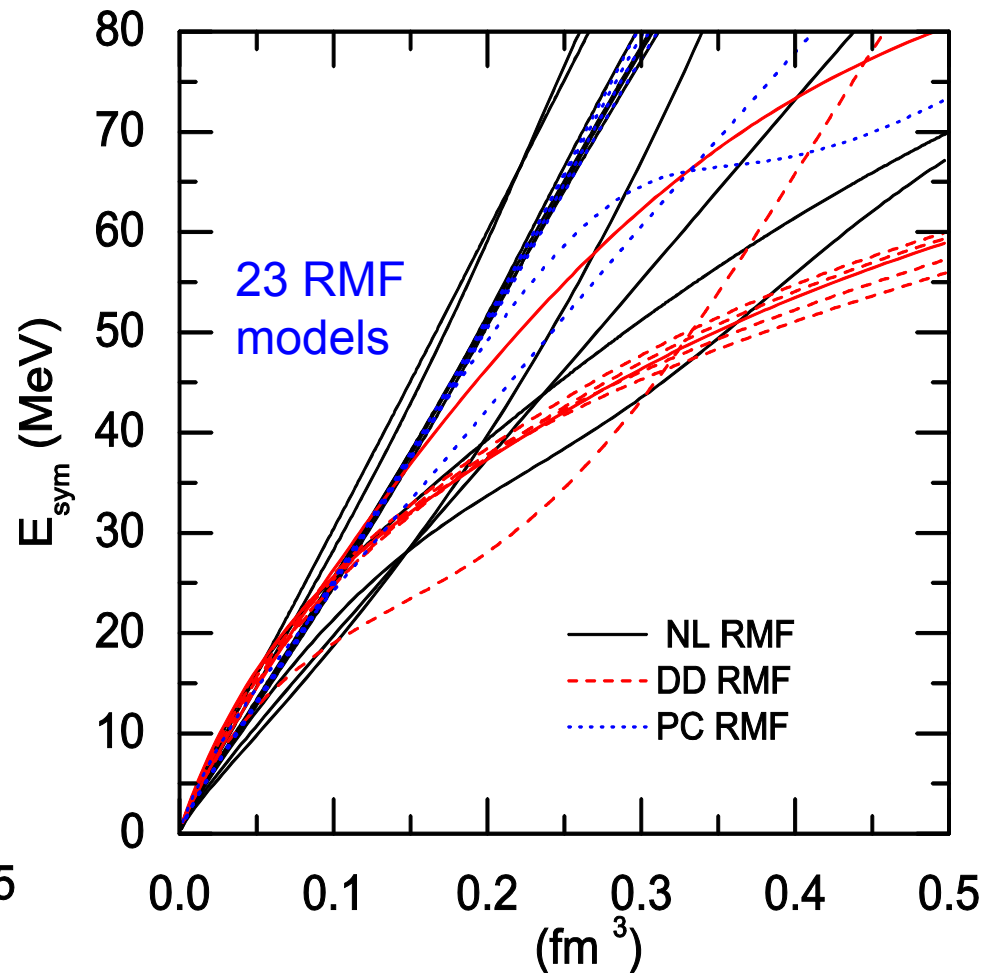
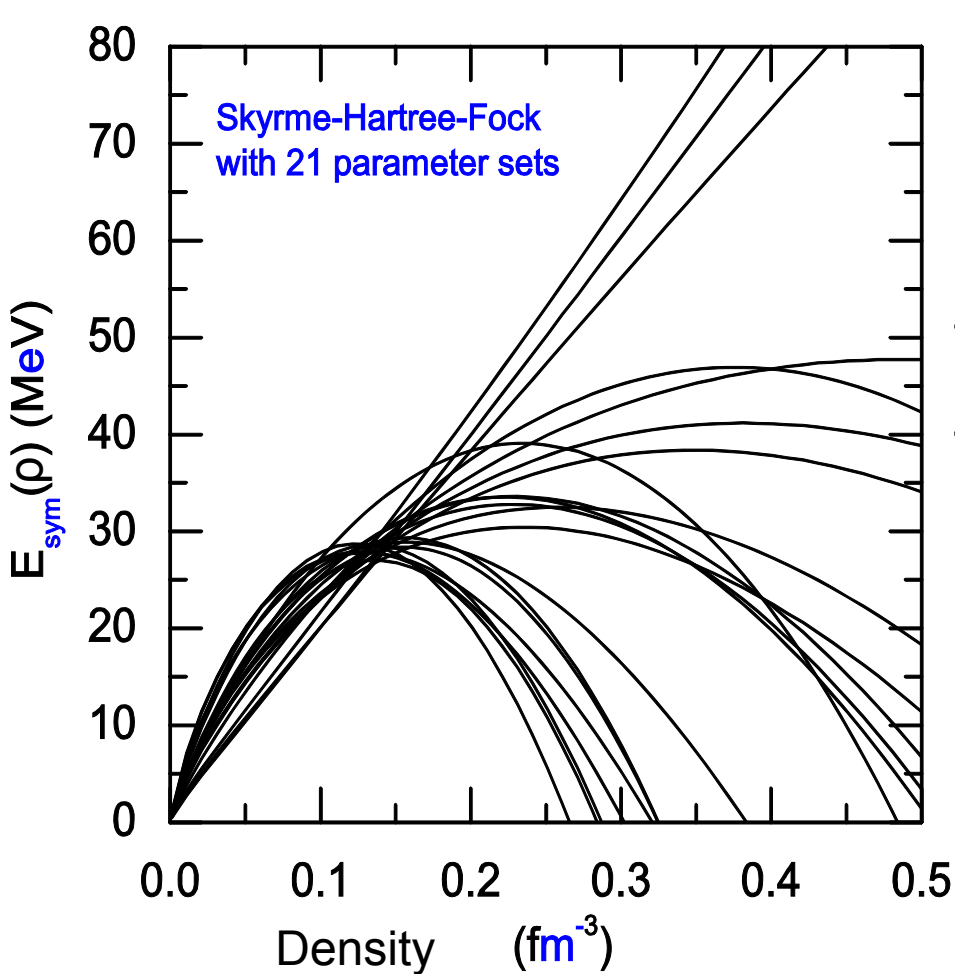
Energy per nucleon in symmetric matter

Energy per nucleon in asymmetric matter

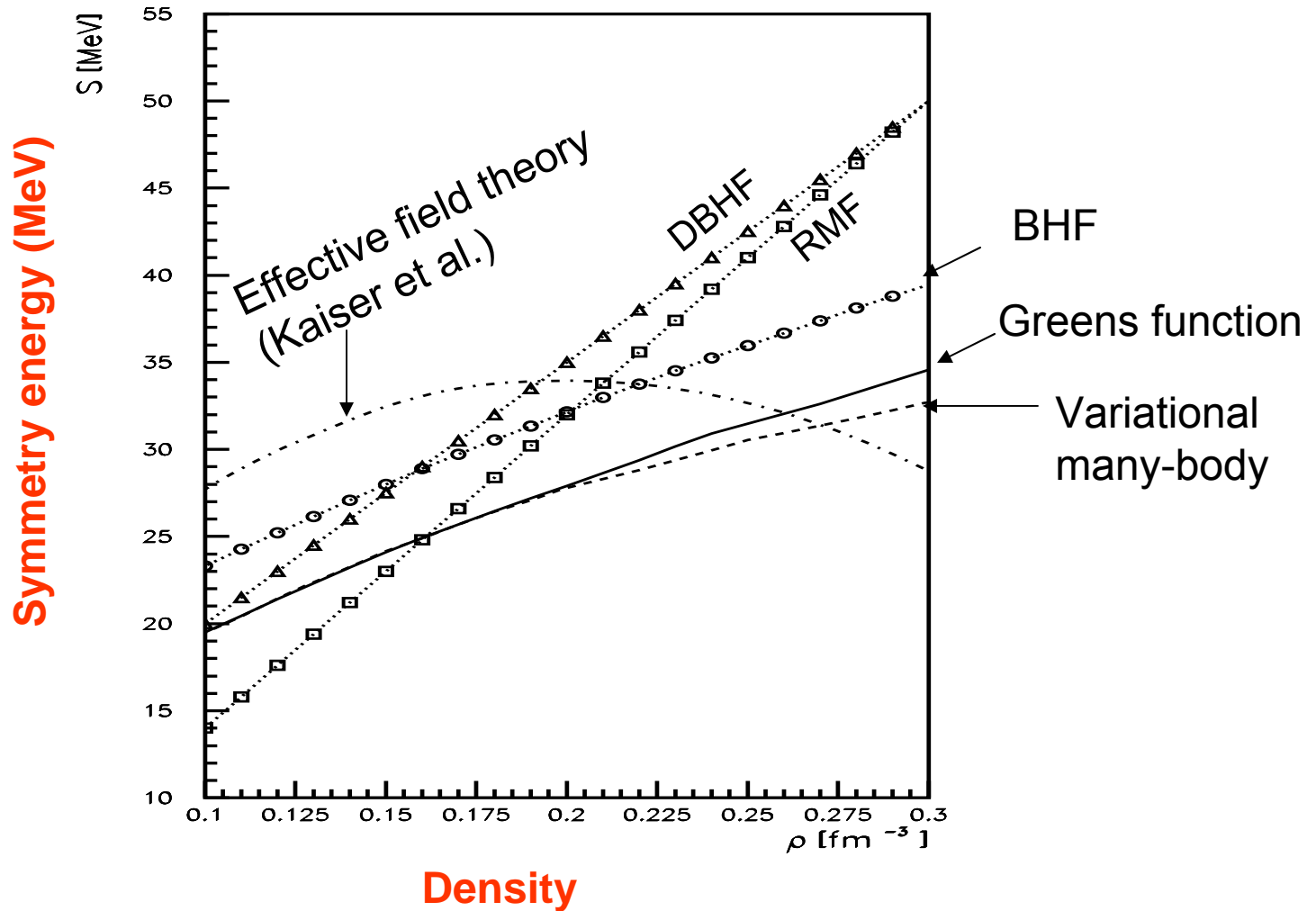


# The $E_{\text{sym}}(\rho)$ from model predictions using popular interactions

Examples:



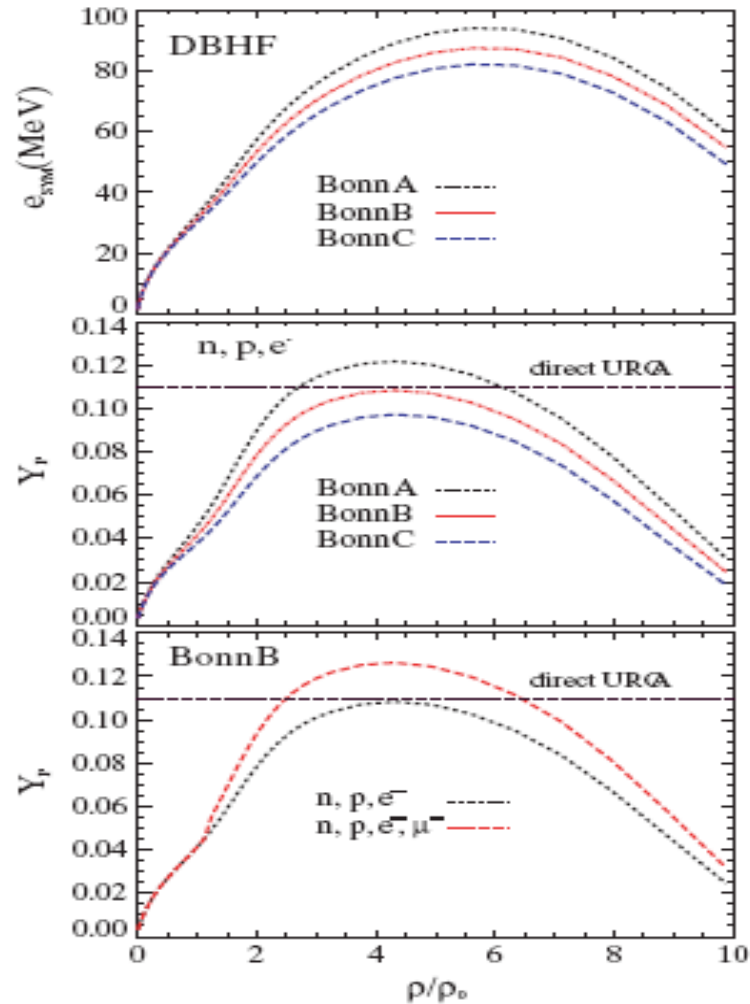
# $E_{\text{sym}}(\rho)$ predicted by microscopic many-body theories



# Dirac-Brueckner-Hartree-Fock Calculations

P. G. KRASDEV AND F. SAMMARRUCA

PHYSICAL REVIEW C 74, 025808 (2006)



# Can the symmetry energy become negative at high densities?

Yes, for example, due to the isospin-dependence of the nuclear tensor force

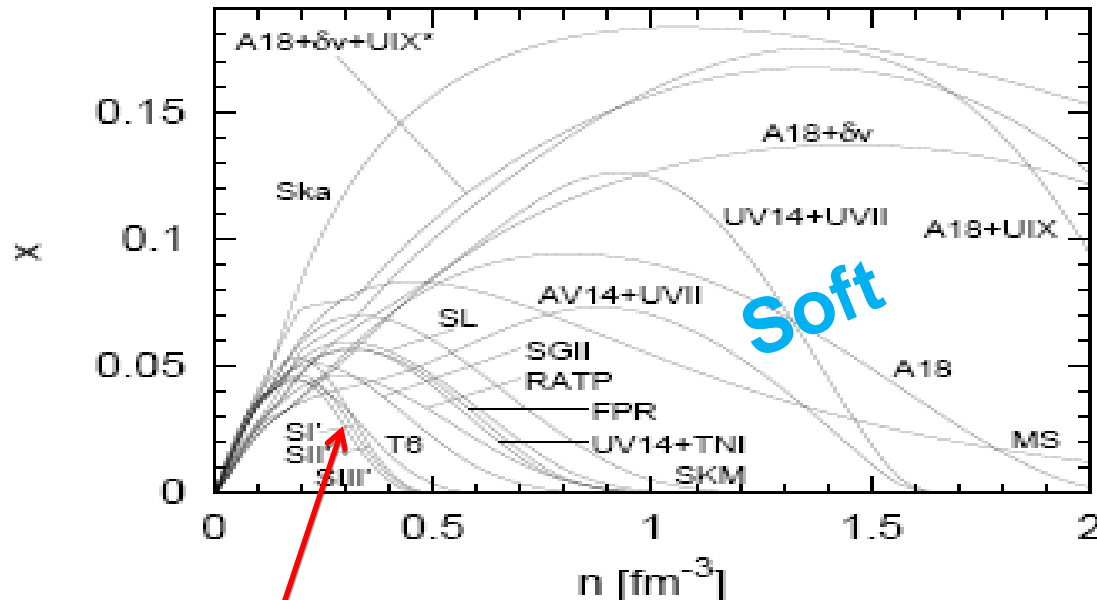
At high densities, the energy of pure neutron matter can be lower than symmetric matter leading to negative symmetry energy

Pandharipande V R and Garde V K 1972 *Phys. Lett. B* 39 608

Wiringa R B, Fiks V and Fabrocini A 1988 *Phys. Rev. C* 38 1010

Kutschera M 1994 *Phys. Lett. B* 340 1

Example: proton fractions with interactions/models leading to negative symmetry energy



**Super-Soft**

**Soft**

ACTA PHYSICA POLONICA B

Vol. 37 (2006)

## PROPERTIES OF LOCALIZED PROTONS IN NEUTRON STAR MATTER FOR REALISTIC NUCLEAR MODELS\*

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$$x = 0.048 [E_{sym}(\rho) / E_{sym}(\rho_0)]^3 (\rho / \rho_0) (1 - 2x)^3$$

# Neutron star and $\beta$ -stable ring-diagram equation of state

Huan Dong and T. T. S. Kuo

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R. Machleidt

*Department of Physics, University of Idaho, Moscow, Idaho 83844, USA*

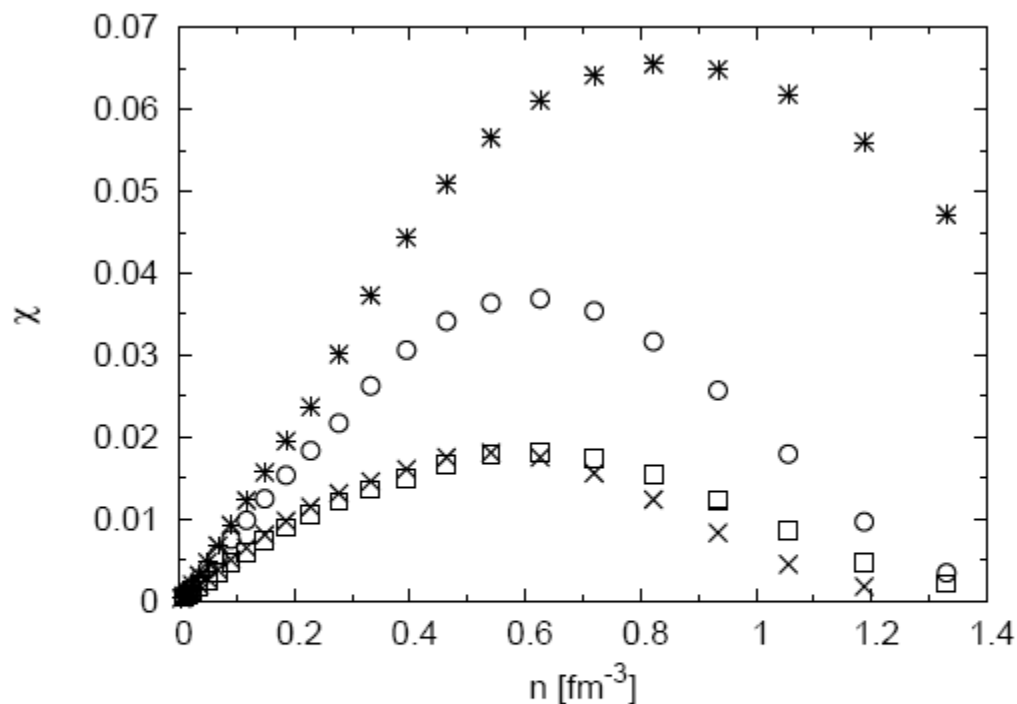


FIG. 9: Proton fraction of  $\beta$ -stable neutron star from realistic NN potentials. Symbols are BonnA(\*), CDBonn(o), Argonne V18 (□) and Nijmegen (x). The interaction ' $V_{low-k}$  plus TBF' is used.

# What are the most important underlying physics determining the symmetry energy at high densities?

Based on the Fermi gas model (Ch. 6) and properties of nuclear matter (Ch. 8) of the textbook:  
*Structure of the nucleus by M.A. Preston and R.K. Bhaduri (1975)*

$$E_{sym} = E_{sym}^{kin} + E_{sym}^{pot1} + E_{sym}^{pot2}$$

$$= \frac{1}{3}t(k_F) + \frac{1}{6} \frac{\partial U_0(k)}{\partial k} \Big|_{k_F} k_F + \frac{3}{2k_F^3} \int_0^{2\frac{1}{3}k_F} U_{sym}(k) k^2 dk$$

Kinetic                      Isoscalar                      Isovector

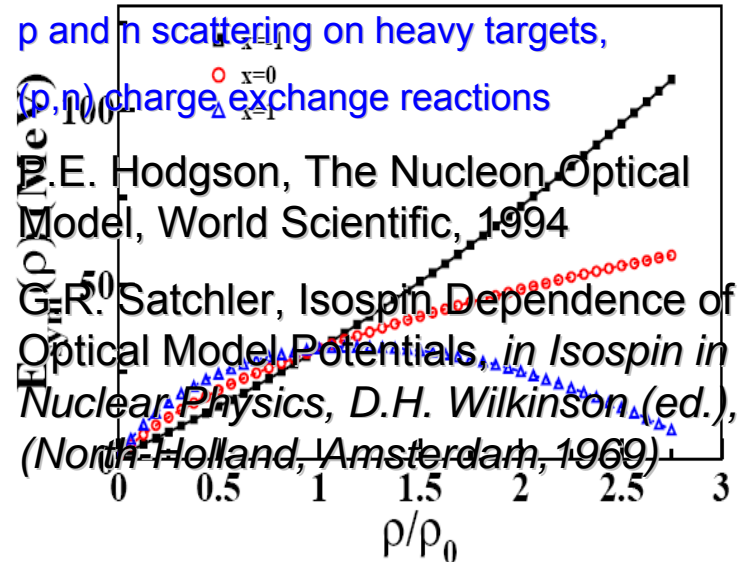
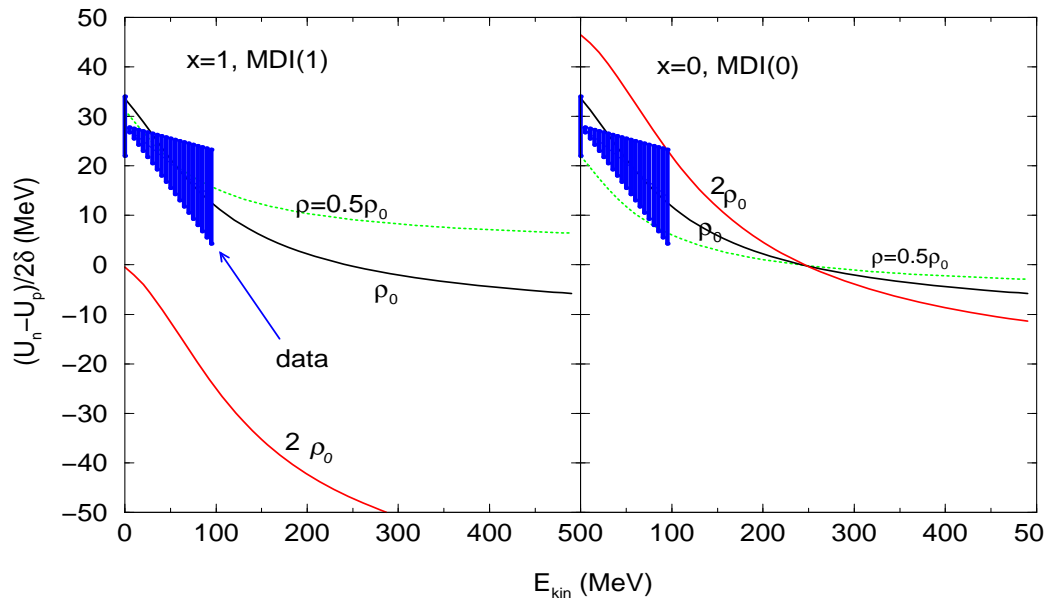
$$U_{n/p} = U_0 \pm U_{sym} \delta$$

Our knowledge of the isovector potential is very poor!

$$U_n = u_{nn} \frac{\rho_n}{\rho} + u_{np} \frac{\rho_p}{\rho} = u_{T1} \frac{\rho_n}{\rho} + u_{T1} \frac{\rho_p}{2\rho} + u_{T0} \frac{\rho_p}{2\rho}$$

$$U_p = u_{pp} \frac{\rho_p}{\rho} + u_{pn} \frac{\rho_n}{\rho} = u_{T1} \frac{\rho_p}{\rho} + u_{T1} \frac{\rho_n}{2\rho} + u_{T0} \frac{\rho_n}{2\rho}$$

isovector potential dependence very poor!



Gogny-HF prediction: C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).



# Symmetry energy and the isospin-dependence of strong interaction

$$\begin{aligned} E_{sym} &= E_{sym}^{kin} + E_{sym}^{pot1} + E_{sym}^{pot2} \\ &= \frac{1}{3}t(k_F) + \frac{1}{6} \frac{\partial U_0(k)}{\partial k} \Big|_{k_F} k_F + \frac{3}{2k_F^3} \int_0^{2^{\frac{1}{3}}k_F} U_{sym}(k)k^2 dk \end{aligned}$$

In coordinate space, in terms of two-body interactions,

$$E_{sym}^{pot2} = \frac{1}{8}\rho \int f_{cor}(r) [V_{T1}(r) - V_{T0}(r)] d^3r$$

Correlation funct:  $f_{cor}(r) = 1 - \left(\frac{3J_1(k_F r)}{k_F r}\right)^2$  in the Fermi gas;

or  $f_{cor}(r) = 0/1$  for  $r$  smaller/larger than a “healing radius” depending on the density;

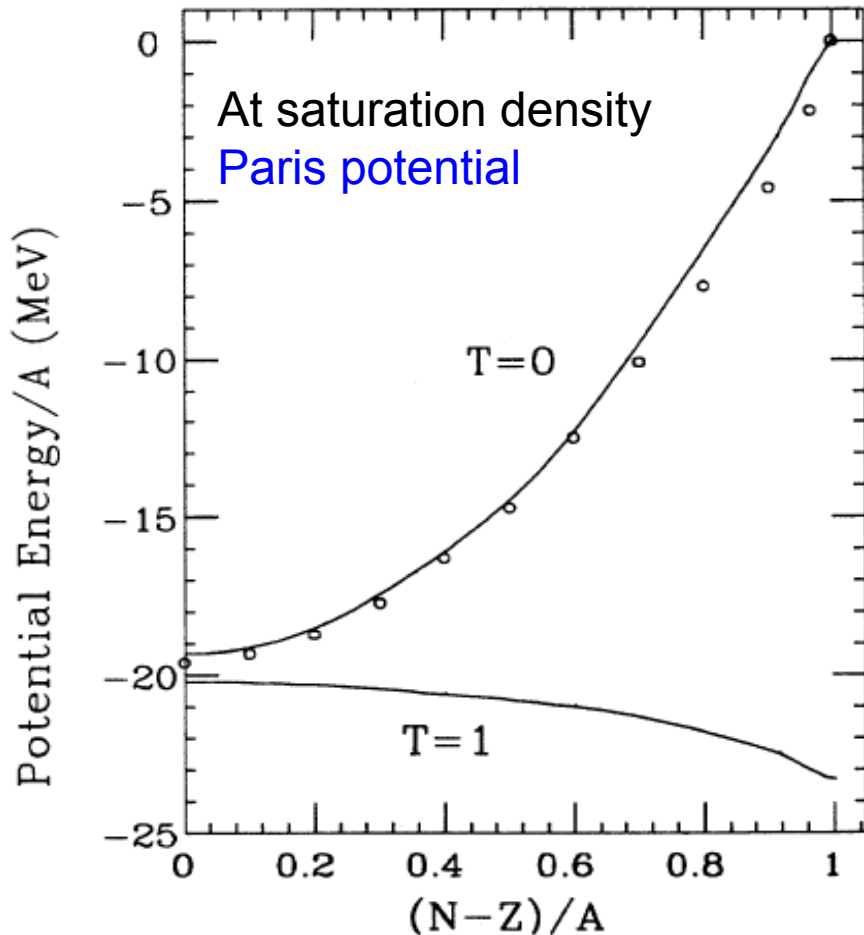
more advanced approaches depending on (S,T) and the interactions ....  $r_c = \eta\left(\frac{3}{4\pi\rho}\right)^{1/3}$

**We are probing the in-medium isospin-dependence of strong interaction**

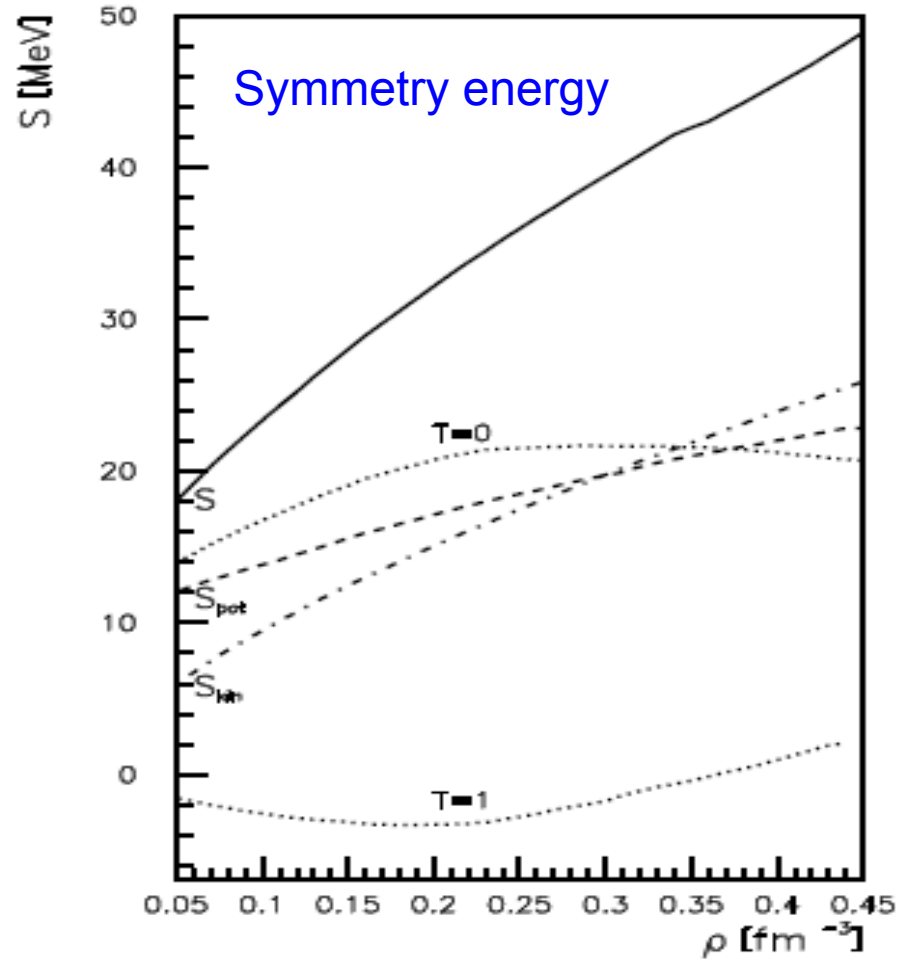
$$V_{T0} = V'_{np} \quad (\text{n-p pair in the } T=0 \text{ state})$$

$$V_{T1} = V_{nn} = V_{pp} = V_{np} \quad (\text{charge independence in the } T=1 \text{ state})$$

# Dominance of the isosinglet (T=0) interaction



I. Bombaci and U. Lombardo PRC 44, 1892 (1991)



A.E.L. Dieperink,<sup>1</sup> Y. Dewulf,<sup>2</sup> D. Van Neck,<sup>2</sup> M. Waroquier,<sup>2</sup> and V. Rodin<sup>3</sup>

PRC68, 064307 (2003)

$$E_{sym}(\rho) = \frac{1}{2} \frac{\partial^2 E}{\partial \delta^2} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

## Parametrization of the Paris $N$ - $N$ potential

M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau

*Division de Physique Théorique, Institut de Physique Nucléaire, Orsay 91406, France  
and LPTPE, Université Pierre et Marie Curie, Paris 75230, France*

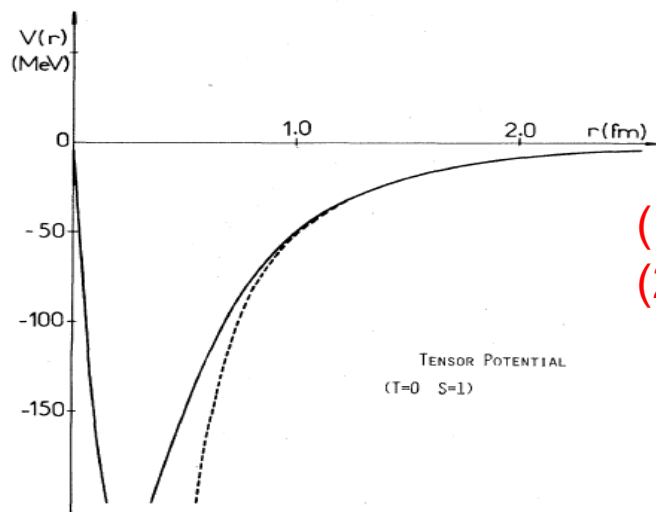
J. Côté, P. Pirès, and R. de Turreil

*Division de Physique Théorique, Institut Physique de Nucléaire, Orsay 91406, France*

(Received 27 July 1979)

small values of  $r$ , there is no compelling theoretical reason to believe the validity of our potential in the region  $r \leq 0.8$  fm since the short range (SR) part of the interaction is related to exchange of heavier systems and/or to effects of subhadronic constituents such as quarks, gluons, etc. At pres-

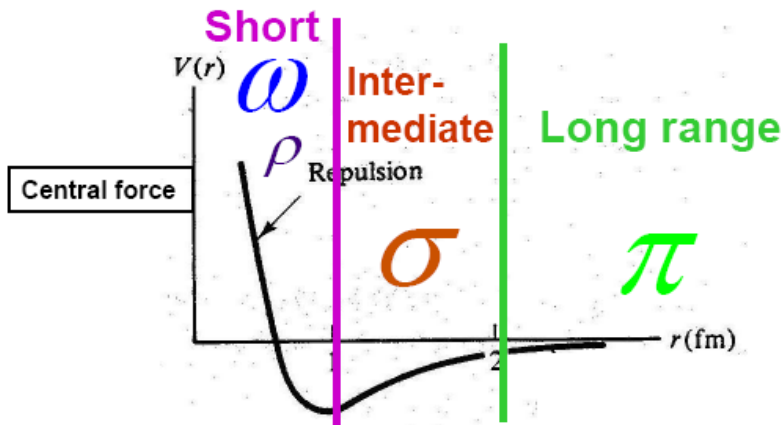
few degrees of freedom. Along this line, we proposed<sup>3</sup> to describe the core with a very simple phenomenological model; namely, the long and intermediate range ( $\pi+2\pi+\omega$ ) potential is cut off rather sharply at internucleon distance  $r \sim 0.8$  fm and the short range ( $r \leq 0.8$  fm) is described simply by a constant soft core. This introduces the



(1) including only pion contribution to the tensor force  
(2) using a hard-core cut-off distance of 0.8 fm

# The short and long range tensor force

Lecture notes of R. Machleidt  
at the 2005 RIKEN summer school



Tensor force

$\pi$   $\rho$

Spin-orbit force

$\omega$   $\sigma$

$\pi$  (138)

$$V_{\pi} = \frac{f_{\pi}^2 NN}{3m_{\pi}^2} \frac{\vec{q}^2}{\vec{q}^2 + m_{\pi}^2} [-\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\vec{q})] \vec{r}_1 \cdot \vec{r}_2$$

Long-ranged  
tensor force

$\sigma$  (600)

$$V_{\sigma} \approx \frac{g_{\sigma}^2}{\vec{q}^2 + m_{\sigma}^2} \left[ -1 - \frac{\vec{L} \cdot \vec{S}}{2M^2} \right]$$

intermediate-ranged,  
attractive central force  
plus LS force

$\omega$  (782)

$$V_{\omega} \approx \frac{g_{\omega}^2}{\vec{q}^2 + m_{\omega}^2} \left[ +1 - 3 \frac{\vec{L} \cdot \vec{S}}{2M^2} \right]$$

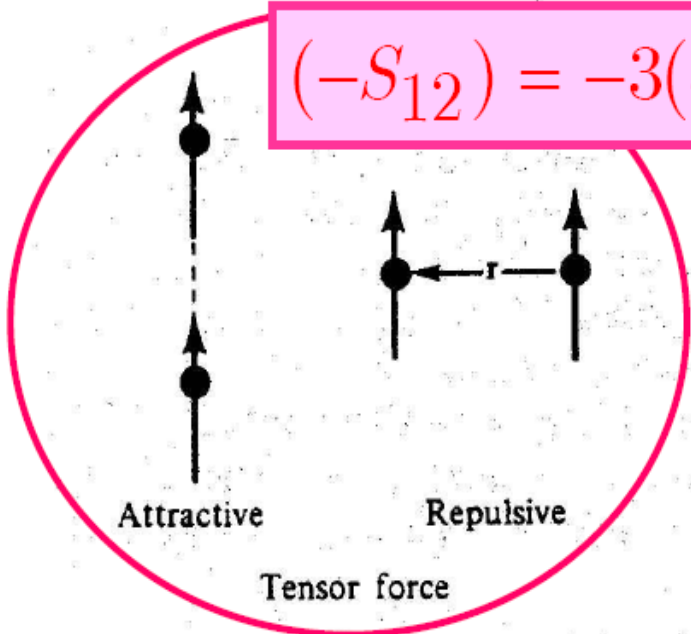
short-ranged,  
repulsive central force  
plus strong LS force

$\rho$  (770)

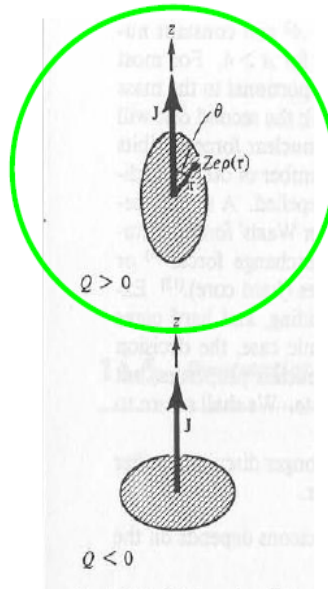
$$V_{\rho} = \frac{f_{\rho}^2}{12M^2} \frac{\vec{q}^2}{\vec{q}^2 + m_{\rho}^2} [-2\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\vec{q})] \vec{r}_1 \cdot \vec{r}_2$$

short-ranged  
tensor force,  
opposite to pion

$$(-S_{12}) = -3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



**Tensor Force: First evidence from the deuteron**



**Deuteron**

$$S=1, T=0, S_{12}=2$$

# In-medium properties of the short-range tensor force

$$V_T^\rho(r) = \frac{f_{N\rho}^2 m_\rho}{4\pi} \tau_1 \cdot \tau_2 \left( S_{12} \left[ \frac{e^{-m_\rho r}}{(m_\rho r)^3} + \frac{e^{-m_\rho r}}{(m_\rho r)^2} + \frac{e^{-m_\rho r}}{3m_\rho r} \right] \right)$$

G.E. Brown and Mannque Rho,  
PLB 237, 3 (1990)

$$V_T^\pi(r) = \frac{f_{N\pi}^2 m_\pi}{4\pi} \tau_1 \cdot \tau_2 \left( -S_{12} \left[ \frac{e^{-m_\pi r}}{(m_\pi r)^3} + \frac{e^{-m_\pi r}}{(m_\pi r)^2} + \frac{e^{-m_\pi r}}{3m_\pi r} \right] \right).$$

## Shell Model Description of the $^{14}\text{C}$ Dating $\beta$ Decay with Brown-Rho-Scaled $NN$ Interactions

J. W. Holt,<sup>1</sup> G. E. Brown,<sup>1</sup> T. T. S. Kuo,<sup>1</sup> J. D. Holt,<sup>2</sup> and R. Machleidt<sup>3</sup> **PRL 100**, 062501 (2008)

A 15% reduction of the rho mass is needed  
to reproduce the right lifetime of  $^{14}\text{C}$

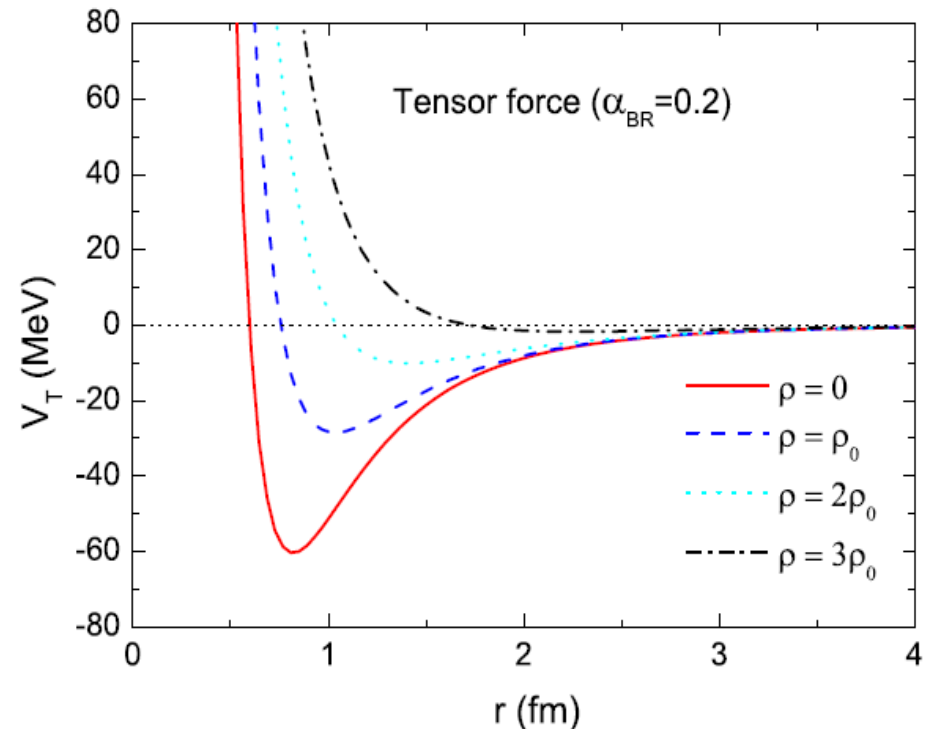
Brown-Rho scaling (BRS)

$$\frac{m_\rho^*}{m_\rho} = 1 - \alpha_{BR} \cdot \frac{\rho}{\rho_0}$$

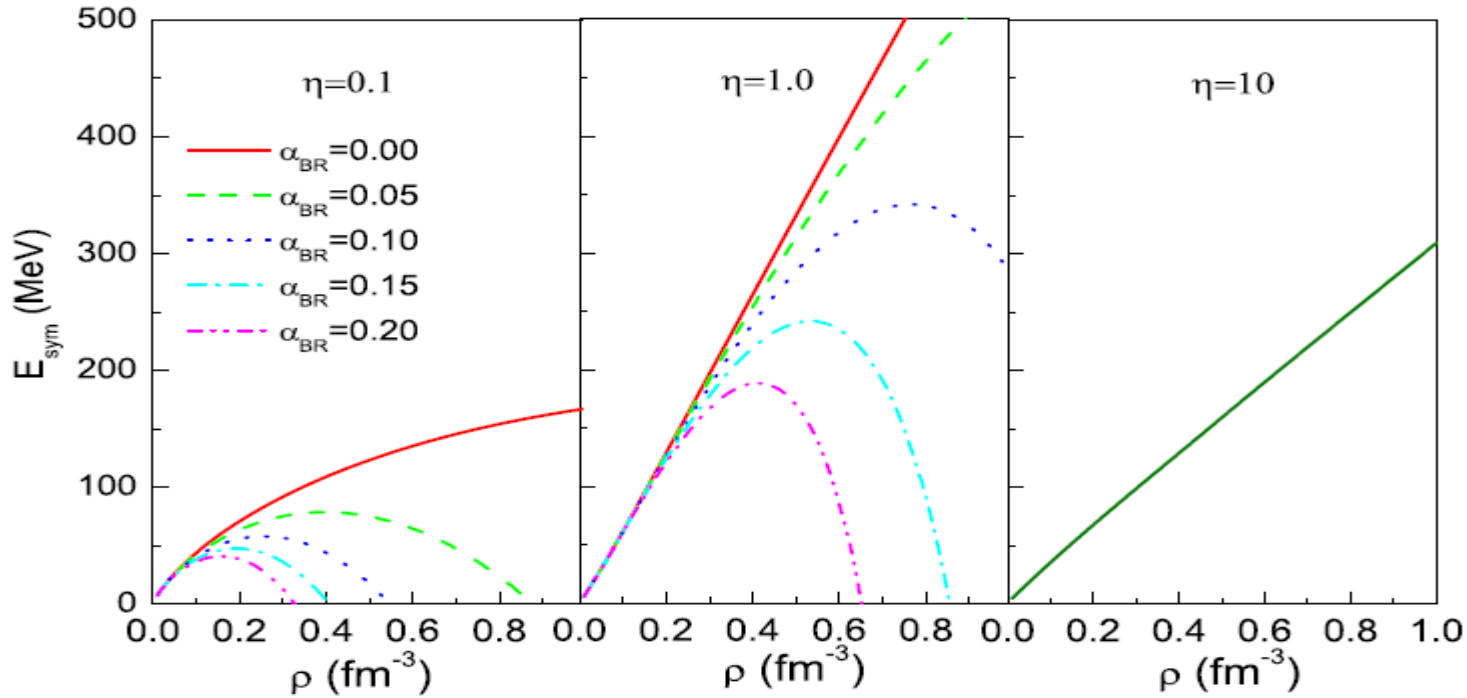
Pion mass unchanged

G.E. Brown and Mannque Rho,  
PRL 66, 2720 (1991); Phys Rep. 396, 1 (2004)

## Strength of the total tensor force



# Effects of the tensor force and short-range correlations on the HdEsym



Assuming  $S_{12}=2$  in  $T=0, S=1$  channel as in deuteron

$$\left(\widetilde{V}_{T0}\right)_{\text{tensor}} = \int f(r_{ij}) [V_T^\rho(r_{ij}) + V_T^\pi(r_{ij})] d^3 r_{ij}$$

$$f(r_{ij}) = 0, r_{ij} < r_c \text{ and } f(r_{ij}) = 1, r_{ij} \geq r_c \text{ with } r_c = \eta \left(\frac{3}{4\pi\rho}\right)^{1/3}$$

$$U_0(k) = \frac{c\rho/\rho_0}{1 + (k/(\Lambda k_F))^2}$$

Gale-Bertsch-Das Gupta's parameterization of the momentum-dependent part of the isoscalar potential

$$V(r) = \sum_{i=1,2} (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau)_i e^{-r^2/\mu_i^2}$$

Gogny central force

# The Gogny force:

$$v(r) = \sum_{i=1,2} (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau)_i e^{-r^2/\mu_i^2} \quad \text{Central}$$

$$+ t_0(1 + P_\sigma) \rho^\alpha \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2). \quad \text{Reduced 3-body force}$$

ST	00	01	10	11
$4\sqrt{\pi} A_i^{(ST)}$	$(W + H - B - M)_i$	$3(W + M - B - H)_i$	$3(W + M + B + H)_i$	$9(W + B - H - M)_i$
$B_i^{(ST)}$	$A_i^{00}$	$-A_i^{01}$	$-A_i^{10}$	$A_i^{11}$
$C^{(ST)}$	0	$\frac{(1-x_0)}{2} \frac{3t_0}{8}$	$\frac{(1+x_0)}{2} \frac{3t_0}{8}$	0

The values of the parameters for  $D1$  and  $D1'$

Range fm	$W$	$B$	$H$	$M$ MeV	$\alpha$	to MeV fm <sup>4</sup>	$x_0$
0.7	-402.4	-100.0	-496.2	-23.56	$\frac{1}{3}$	1350	1
1.2	-21.30	-11.77	37.27	-68.81	$\frac{1}{3}$		
$W_{LS}=115$ for $D1$		$W_{LS}=130$ for $D1'$					



# TWO-BODY AND THREE-BODY EFFECTIVE INTERACTIONS IN NUCLEI †

NAOKI ONISHI †† and J. W. NEGELE †††

*Nuclear Physics A*301 (1978) 336–348

D. Vautherin and D.M.Brink, *Phys.Rev.C*5, 626 (1972)

+ MANY other papers starting from the same 3-body force,

$$V_3(\xi_1 \xi_2 \xi_3) = t_3 \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_2 - \mathbf{r}_3)$$

Reduced to different 2-body force with  $\alpha=1/3, 2/3, 1, \text{ etc}$

$$t_0(1 + x_0 P_\sigma) \rho^\alpha \left( \frac{r_1 + r_2}{2} \right) \delta(r_1 - r_2)$$

E. Chabanat <sup>a</sup>, P. Bonche <sup>b</sup>, P. Haensel <sup>c</sup>, J. Meyer <sup>a,1</sup>, R. Schaeffer <sup>b</sup>

*Nuclear Physics A* 627 (1997) 710–746

# Skyrme Interaction parameterizations of two+three body force

**$K$  - kinetic energy term,  $H_0$  zero-range term,  
 $H_3$  - density dependent term,  $H_{eff}$  - effective mass term  
 $n$  - particle number density,  $\tau$  - kinetic energy density  
Dependent on 9 parameters  $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \alpha$**

$$\mathcal{H}_0 = \frac{1}{4}t_0[(2 + x_0)n^2 - (2x_0 + 1)(n_p^2 + n_n^2)],$$

$$\mathcal{H}_3 = \frac{1}{24}t_3n^\alpha[(2 + x_3)n^2 - (2x_3 + 1)(n_p^2 + n_n^2)],$$

$$\begin{aligned}\mathcal{H}_{eff} = & \frac{1}{8}[t_1(2 + x_1) + t_2(2 + x_2)]\tau n \\ & + \frac{1}{8}[t_2(2x_2 + 1) - t_1(2x_1 + 1)](\tau_p n_p + \tau_n n_n)\end{aligned}$$

**D. Vautherin and D.M.Brink, Phys.Rev.C5, 626 1972**

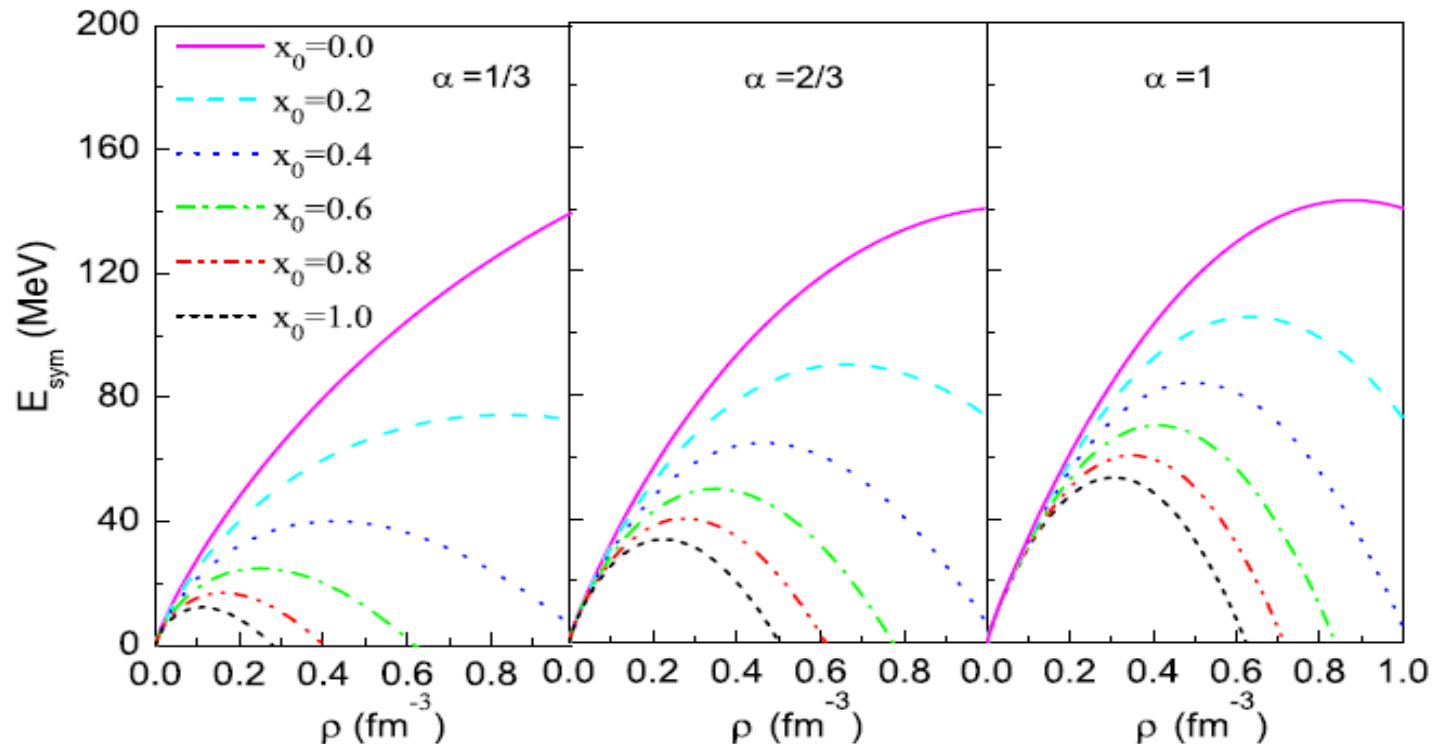
# Effects of the 3-body force on the symmetry energy

The 3-body force contribution to the T=1 and T=0 channel potential energies

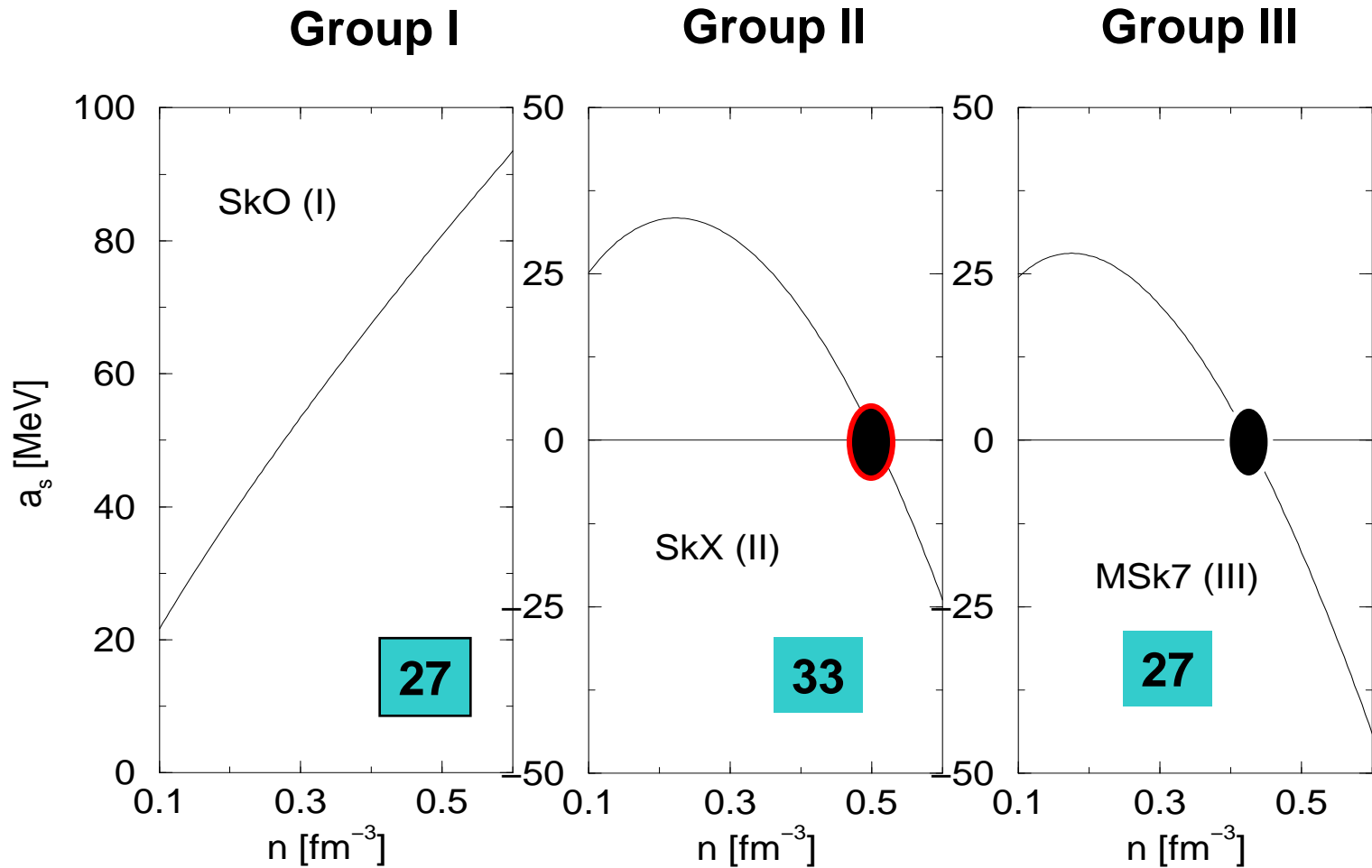
$$E_d^{T1} = \frac{1 - x_0}{2} \frac{3t_0}{8} \rho^{\alpha+1}; \quad E_d^{T0} = \frac{1 + x_0}{2} \frac{3t_0}{8} \rho^{\alpha+1}.$$

The symmetric EOS is NOT affected by the variation of  $x_0$  but  $\alpha$

$$E_{sym}^{pot2} = - \sum_{i=1}^{\infty} \left( \frac{H_i}{4} + \frac{M_i}{8} \right) \pi^{\frac{3}{2}} \mu_i^3 \rho - (1 + 2x_0) \frac{t_0}{8} \rho^{\alpha+1}$$



Density dependence of the symmetry energy is the main criterion for distinction between Skyrme parameterizations (87 tested)



J.R. Stone et al., PRC 68, 034324 (2003)

SkO, SkX and MSk7 are examples of Skyrme potentials

## Some observations:

Why is the symmetry energy so uncertain especially at high densities?

- In-medium properties of the short-range tensor force in the n-p ( $T=0$ ) channel, controlled by the in-medium rho-meson mass
- Isospin-dependence of short-range nucleon-nucleon correlations
- Effects of many-body forces

Can the symmetry energy becomes super-soft or even negative at high densities?

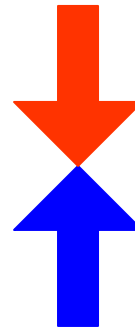
- There is NO first principle forbidding it
- It happens when the repulsive short-range tensor force due to the  $\rho$ -meson exchange in the n-p singlet channel dominates.
- The Lane potential  $U_n - U_p$  flips sign when the symmetry energy starts decreasing with increasing density

**A challenge: how can neutron stars be stable with a super-soft symmetry energy?**

If the symmetry energy is too soft, then a mechanical instability will occur when  $dP/d\rho$  is negative, neutron stars will then all collapse while they do exist in nature

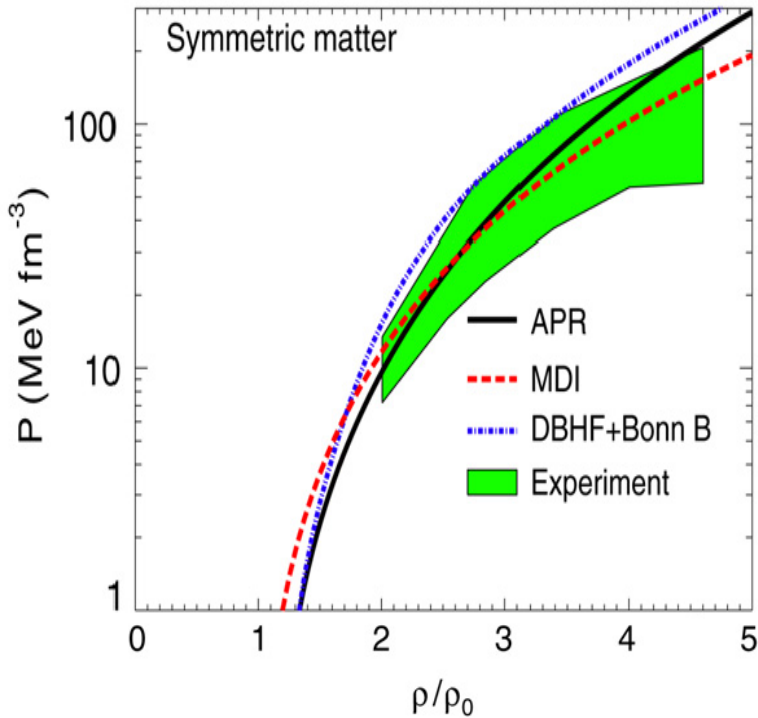
TOV equation: a condition at hydrodynamical equilibrium

$$\frac{dP}{dr} = -(\epsilon + P) \frac{m_g + 4\pi r^3 P}{r(r - 2m_g)}$$



Gravity

Nuclear pressure



For npe matter

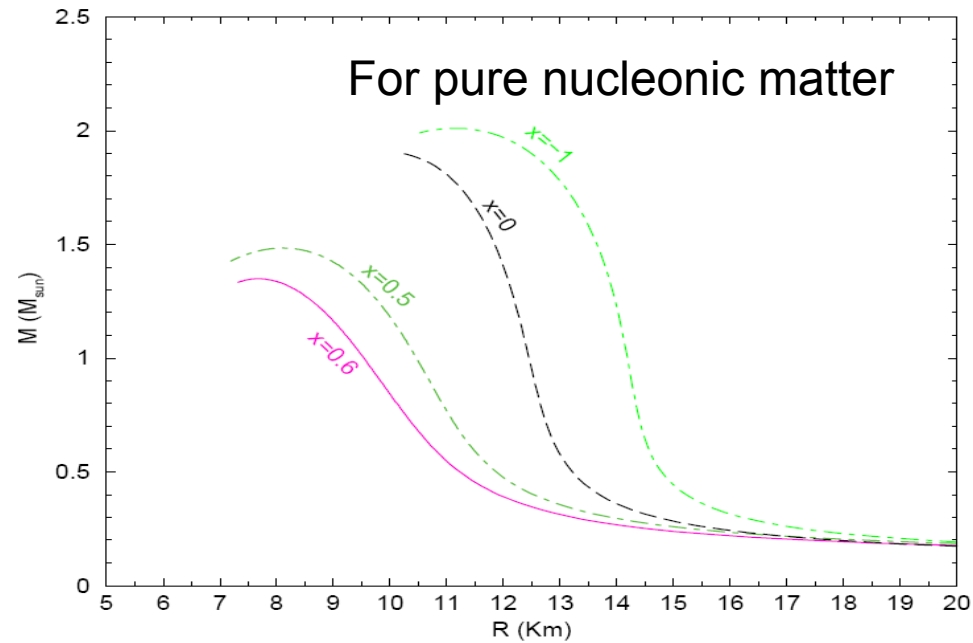
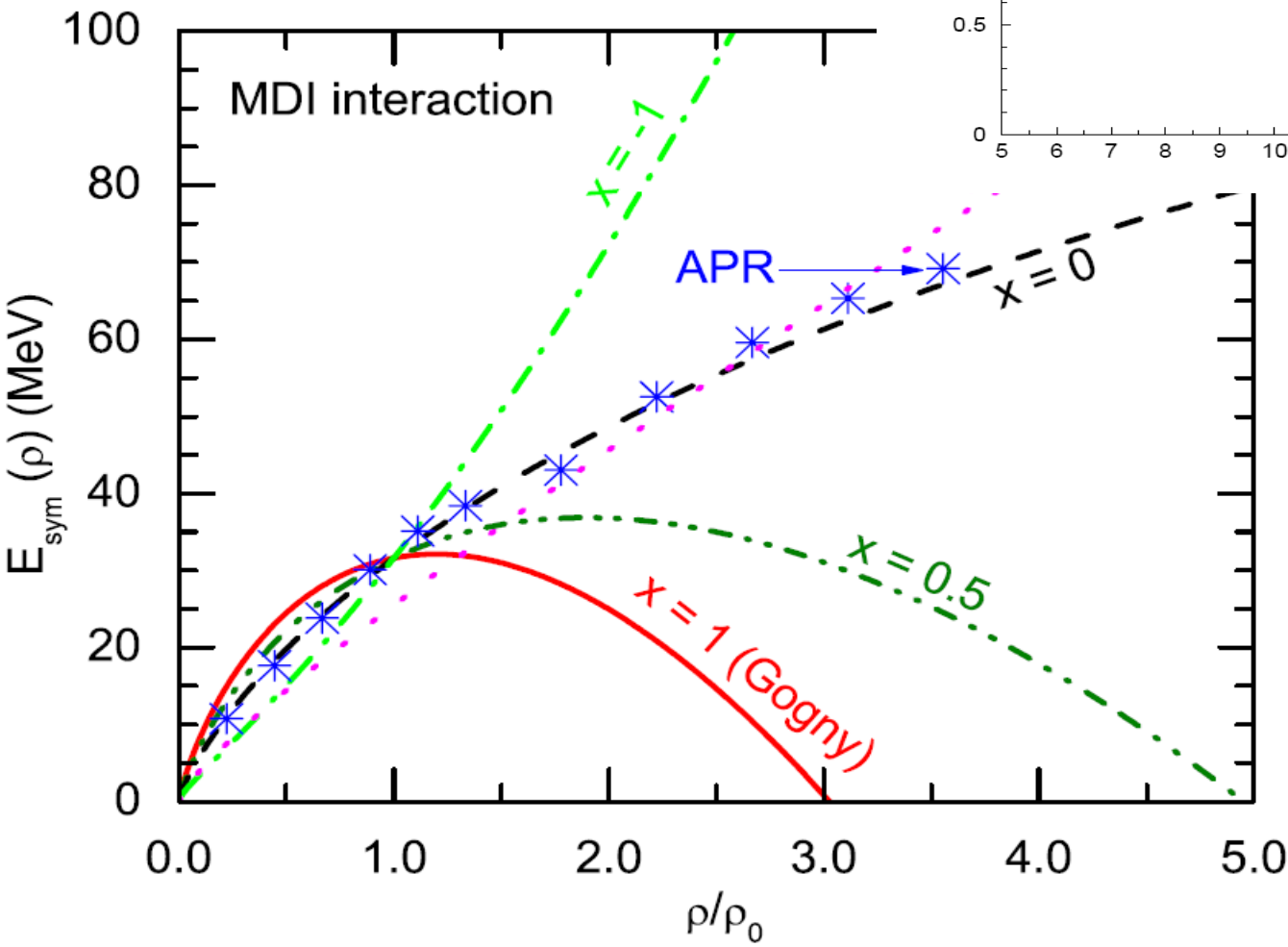
$$P(\rho, \delta) = P_0(\rho) + P_{asy}(\rho, \delta) = \rho^2 \left( \frac{\partial E}{\partial \rho} \right)_\delta + \frac{1}{4} \rho_e \mu_e$$

$$= \rho^2 \left[ E'(\rho, \delta = 0) + E'_{sym}(\rho) \delta^2 \right] + \frac{1}{2} \delta(1 - \delta) \rho E_{sym}(\rho)$$

$dP/d\rho < 0$  if  $E'_{sym}$  is big and negative (super-soft)

P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002))

# Astrophysical implications



Using the EOS for symmetric matter consistent with the terrestrial nuclear reaction data up to  $5 \rho_0$ , the softest symmetry energy that the TOV is still stable is  $x=0.93$  giving  $M_{\text{max}}=0.11$  solar mass and  $R \geq 28$  km.

# Is the super-soft symmetry energy “unpleasant”, “unphysical” or ?

Unpleasant, unwelcome, annoying !

E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer,  
NPA627, 710 (1997); NPA635, 231 (1998).

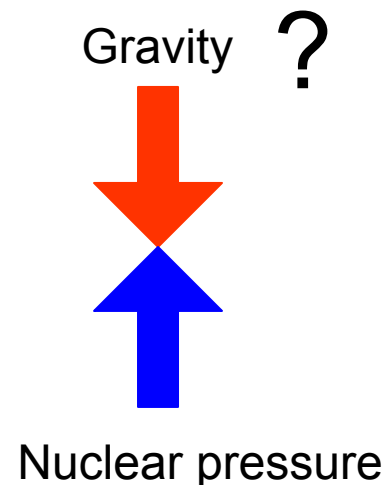
Repeated by several others in some other papers

Unphysical !

Norman Glendinning, Compact Stars, Springer, ISBN: 0387989773.

Quoted by several people in a number of papers

It is unphysical and you are crazy!





# Do we really know gravity well at the Fermi distance?

“It's remarkable that gravity, despite being the first to be discovered, is by far the most poorly understood force”, Roland Pease, *Nature* 411, 986 (2001)

In grand unification theories, conventional gravity has to be modified due to **geometrical effects of extra dimensions at short length, a new boson or the 5<sup>th</sup> force**

$$F(r) = G \frac{m_1 m_2}{r^{2+\epsilon}}$$

String theorists have published TONS of papers on the extra space-time dimensions

N. Arkani-Hamed et al., *Phys Lett. B* 429, 263–272 (1998); J.C. Long et al., *Nature* 421, 922 (2003); C.D. Hoyle, *Nature* 421, 899 (2003)

In terms of the gravitational potential

$$V(r) = -G \frac{m_1 m_2}{r} [1 + \alpha e^{-r/\lambda}]$$

Yukawa potential due to the exchange of a new boson proposed in the super-symmetric extension of the Standard Model of the Grand Unification Theory, or the fifth force

Yasunori Fujii, *Nature* 234, 5-7 (1971); G.W. Gibbons and B.F. Whiting, *Nature* **291**, 636 - 638 (1981)

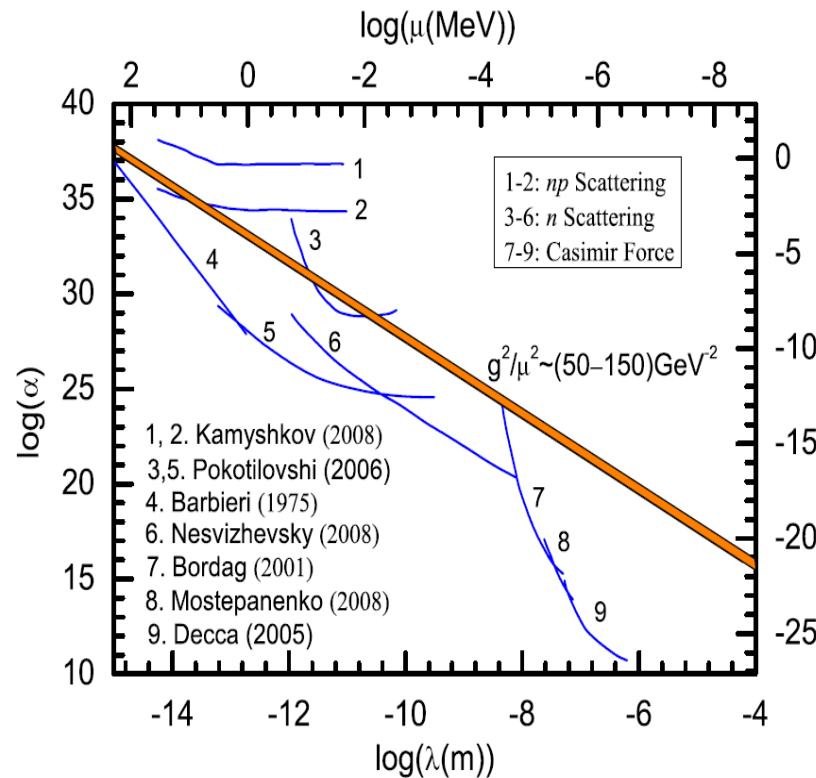
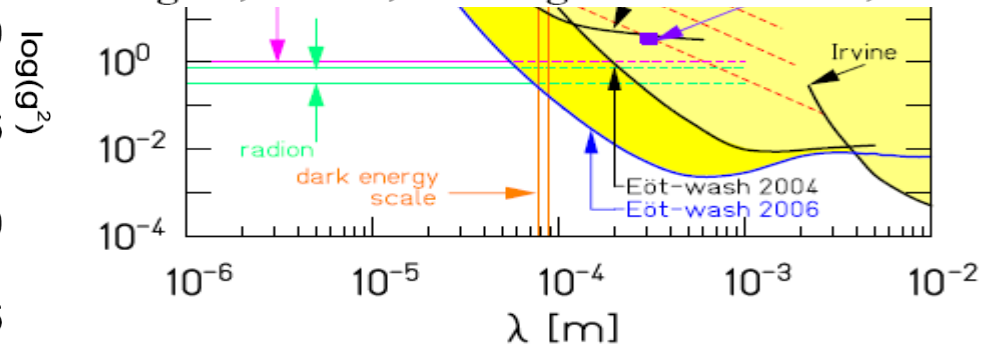
# IONAL

Annu. Rev. Nucl. Part. Sci. 2003. 53:77–121

kel, and A.E. Nelson

Washington, Seattle, Washington 98195-1560;

Torsion  
balance



**Upper limits on the strength  $\alpha$  and range  $\lambda$  of the Yukawa term**

M.I. Krivoruchenko et al., PRD 79, 125023 (2009)

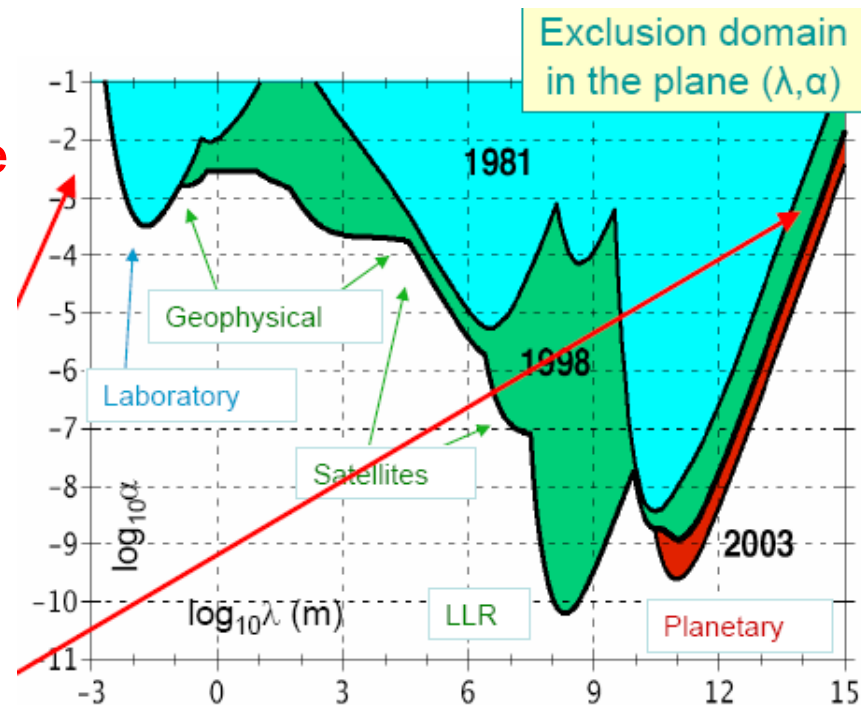
E.G. Adelberger et al., PRL 98, 131104 (2007)

D.J. Kapner et al., PRL 98, 021101 (2007)

Serge Reynaud et al., Int. J. Mod. Phys. A20, 2294 (2005)

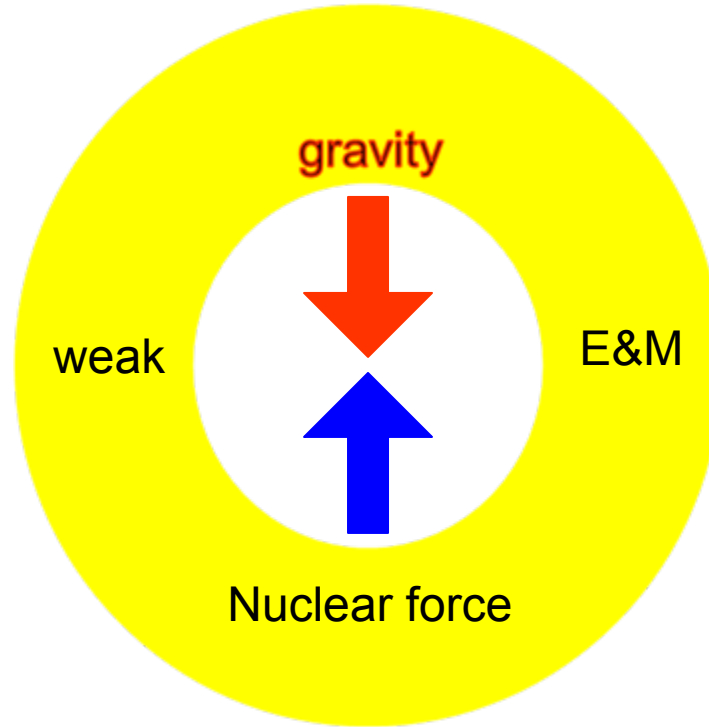
$$V(r) = -G \frac{m_1 m_2}{r} [1 + \alpha e^{-r/\lambda}]$$

$$g^2 = \pm 4\pi G_\infty \mu^2 \alpha \text{ where } \mu = 1/\lambda$$



A motivation of the deep space gravity probe

# Neutron stars as a natural testing ground of grand unification theories of fundamental forces?



Stable neutron star @  $\beta$ -equilibrium

**Requiring simultaneous solutions in both gravity and nuclear force!**  
**Grand Unified Solutions of Fundamental Problems in Nature!**

# Influences of the Yukawa term on Neutron stars

Yasunori Fujii *J. Audouze et al. (eds.), Large Scale Structures of the Universe, 471–477.  
© 1988 by the IAU.*

I next emphasize that the 5-th force is simply part of the matter system in general relativity. Consequently Einstein's equation remains unchanged. The only change one expects to occur is in the equation of state. And probably the first reasonable thing to do is to appeal to the mean field approximation.[11]

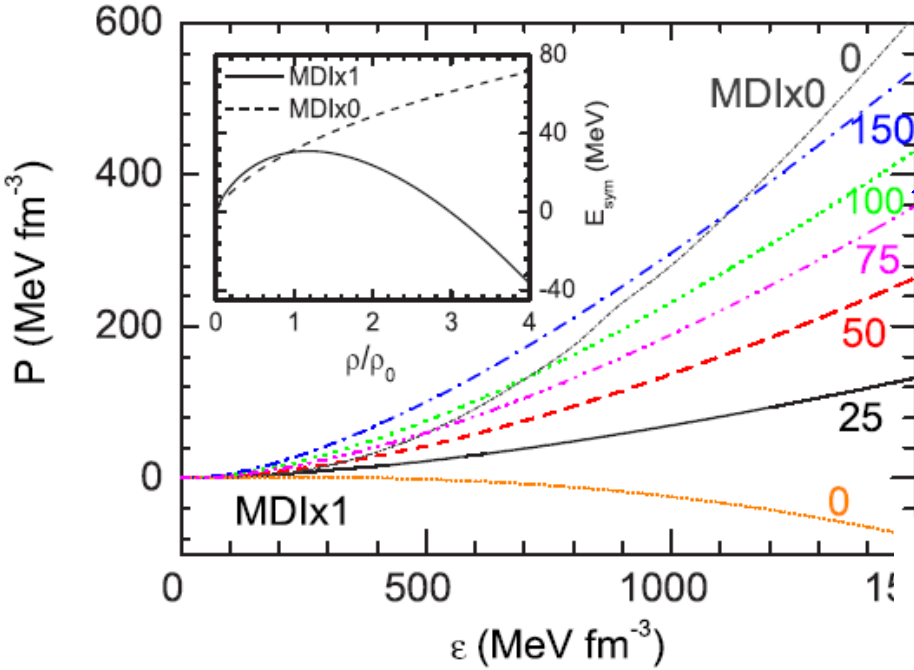
$$\varepsilon_{UB} = \frac{1}{2V} \int \rho(\vec{x}_1) \frac{g^2}{4\pi} \frac{e^{-\mu r}}{r} \rho(\vec{x}_2) d\vec{x}_1 d\vec{x}_2 = \frac{1}{2} \frac{g^2}{\mu^2} \rho^2,$$

$$P_{UB} = \frac{1}{2} \frac{g^2 \rho^2}{\mu^2} \left( 1 - \frac{2\rho}{\mu} \frac{\partial \mu}{\partial \rho} \right).$$

Assuming a constant boson mass independent of the density, the extra pressure is then

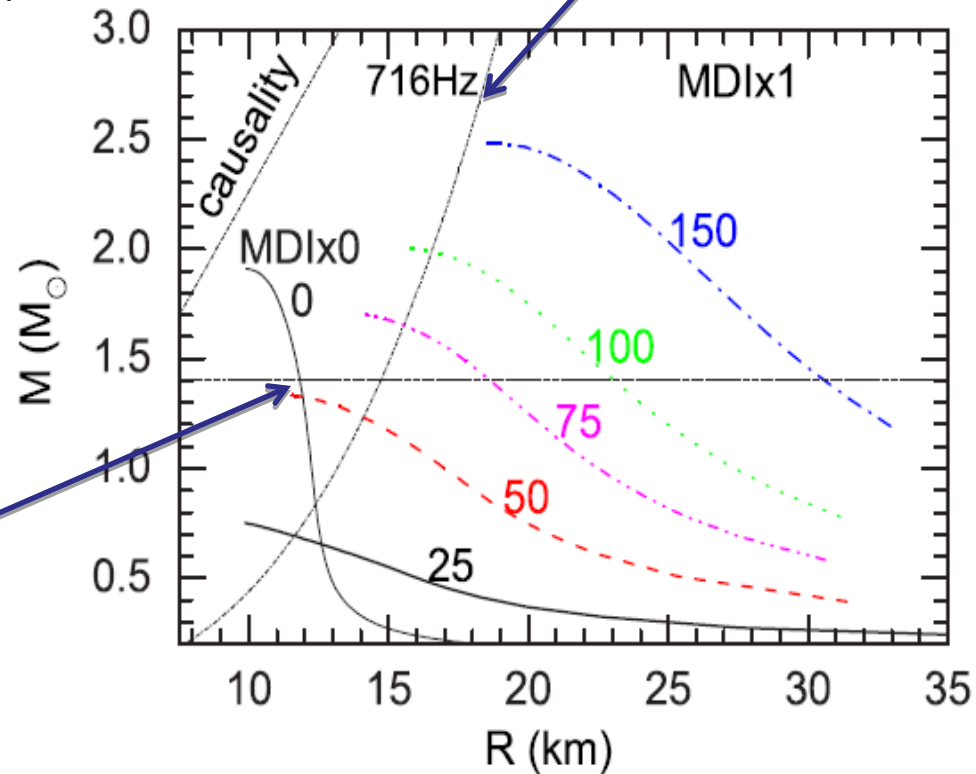
$$P_{UB} = \varepsilon_{UB} = \frac{1}{2} \frac{g^2}{\mu^2} \rho^2 \quad (4)$$

# Nuclear pressure including the Yukawa contribution



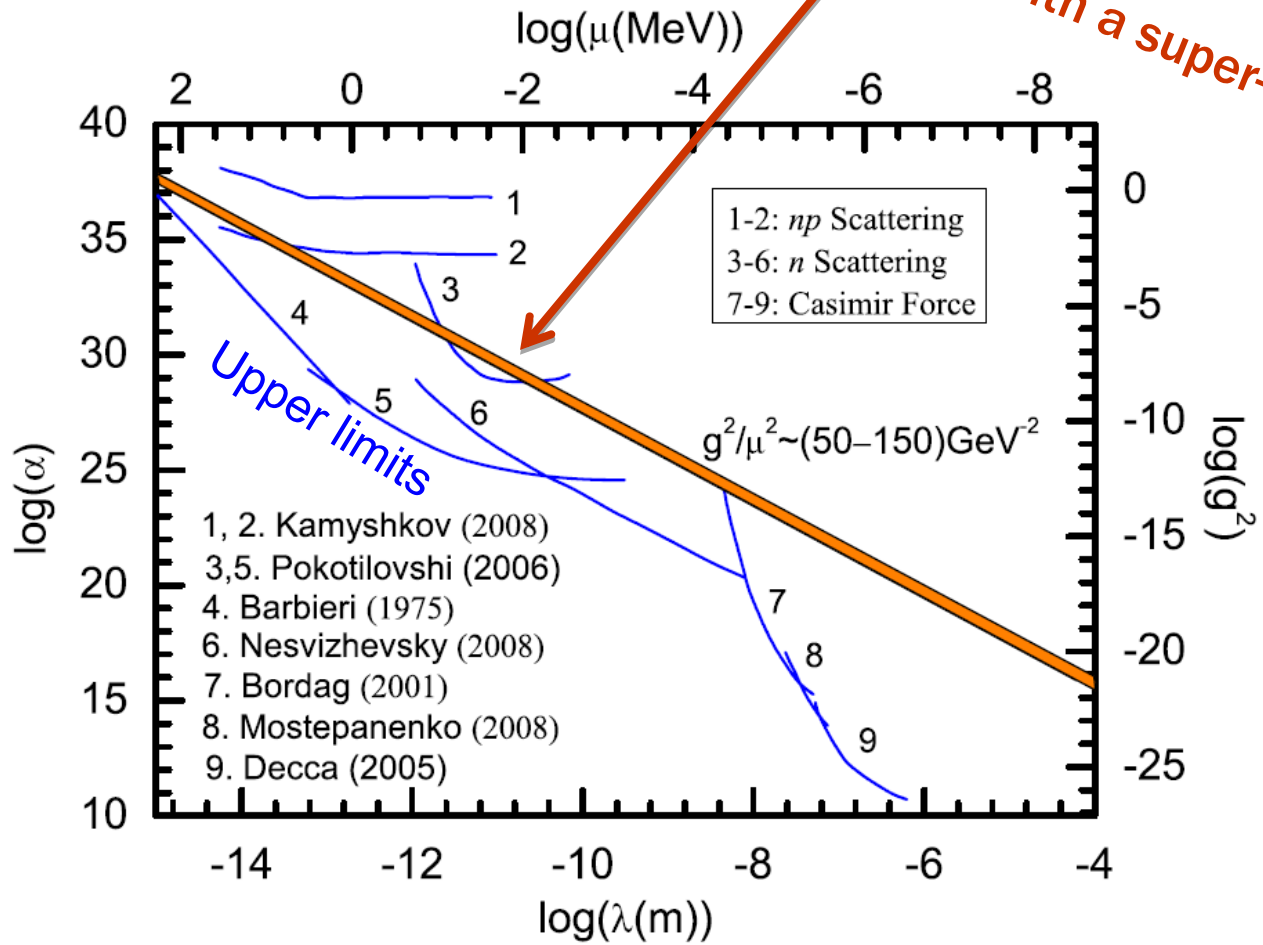
$$g^2 / \mu^2$$

Mass-shedding limit



$g^2 / \mu^2 = 50 \text{ GeV}^{-2}$  to support  
 a NS of  $1.4 M_{\text{sun}}$  and  $R = 12 \text{ km}$

Lower limit to support neutrons stars with a super-soft symmetry energy



# Promising Probes of the $E_{\text{sym}}(\rho)$ in Nuclear Reactions

## At sub-saturation densities

- Sizes of n-skins of unstable nuclei from total reaction cross sections
- **Proton-nucleus elastic scattering in inverse kinematics**
- Parity violating electron scattering studies of the n-skin in  $^{208}\text{Pb}$  at JLab
- **n/p ratio of FAST, pre-equilibrium nucleons**
- Isospin fractionation and isoscaling in nuclear multifragmentation
- **Isospin diffusion/transport**
- Neutron-proton differential flow
- **Neutron-proton correlation functions at low relative momenta**
- $t/{}^3\text{He}$  ratio

## Towards supra-saturation densities

- **$\pi^-/\pi^+$  ratio,  $K^+/K^0$  ?**
- Neutron-proton differential transverse flow
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- Nucleon elliptical flow at high transverse momentum
- $t/{}^3\text{He}$  differential and difference transverse flow

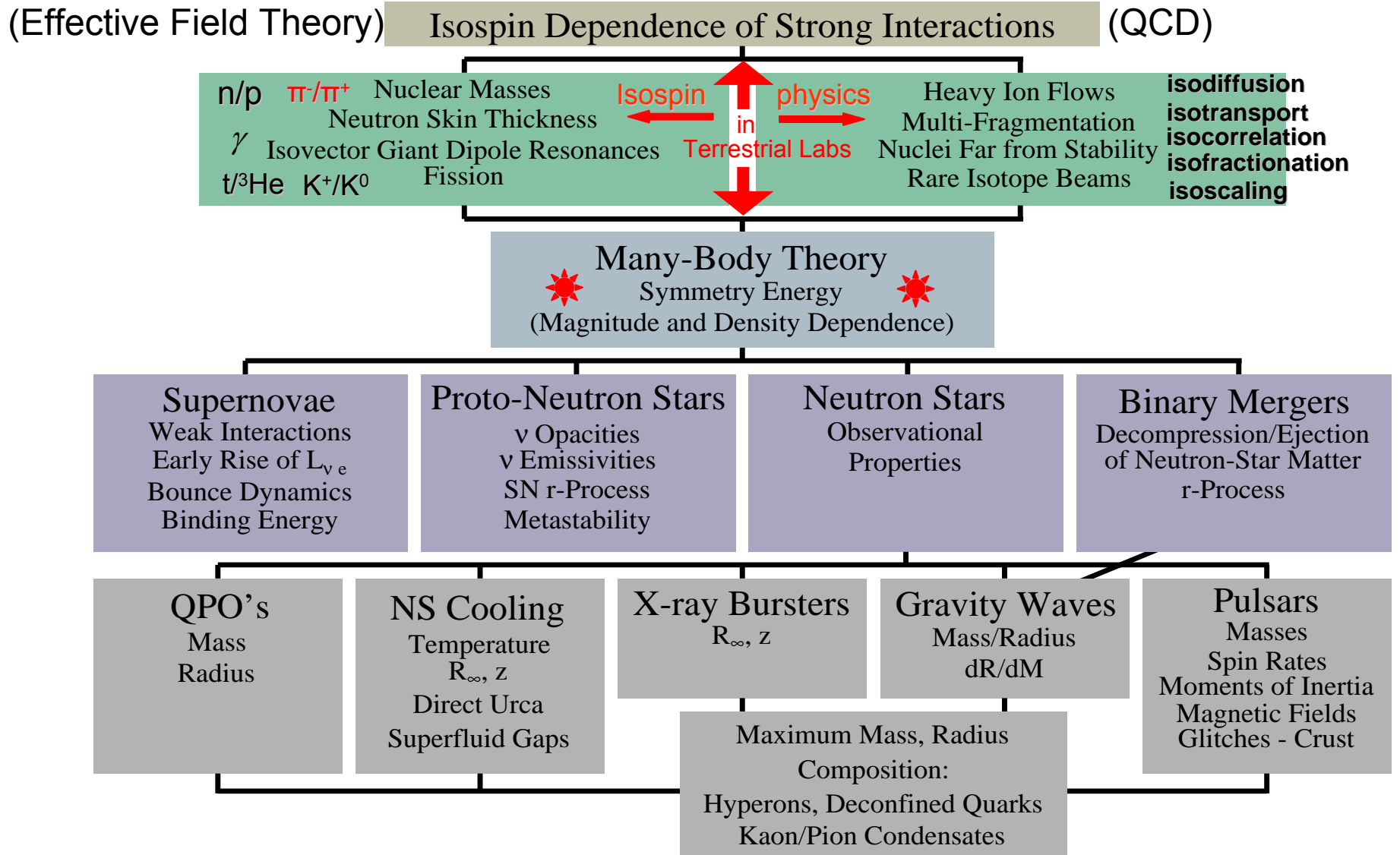
(1) Correlations of multi-observable are important

(2) Detecting neutrons simultaneously with charged particles is critical

# The multifaceted influence of the isospin dependence of strong interaction and symmetry energy in nuclear physics and astrophysics

J.M. Lattimer and M. Prakash, *Science Vol. 304 (2004) 536-542*.

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, *Phys. Rep. 411, 325 (2005)*.





# Pion ratio probe of symmetry energy at supra-normal densities

GC Coefficients <sup>2</sup>	$\pi^+$	$\pi^0$	$\pi^-$
nn	0	1	5
pp	5	1	0
np(pn)	1	4	1

a)  $\Delta(1232)$  resonance model  
in first chance NN scatterings:  
(*neglect rescattering and reabsorption*)

$$\frac{\pi^-}{\pi^+} = \frac{5 N^2 + NZ}{5 Z^2 + NZ} \approx \left( \frac{N}{Z} \right)^2$$

*R. Stock, Phys. Rep. 135 (1986) 259.*

b) **Thermal model:**

(*G.F. Bertsch, Nature 283 (1980) 281; A. Bonasera and G.F. Bertsch, PLB195 (1987) 521*)

$$\frac{\pi^-}{\pi^+} \propto \exp[2(\mu_n - \mu_p) / kT]$$

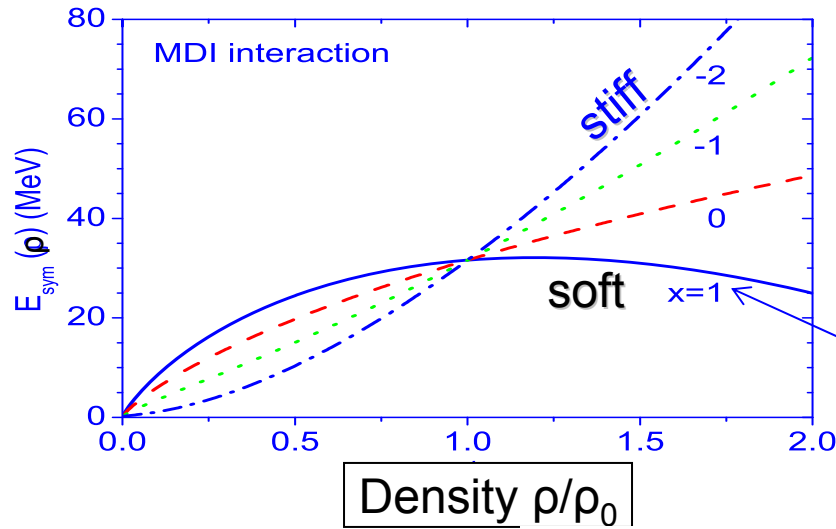
$$\mu_n - \mu_p = (V_{asy}^n - V_{asy}^p) \delta - V_{Coul} + kT \left\{ \ln \frac{\rho_n}{\rho_p} + \sum_m \frac{m+1}{m} b_m \left( \frac{1}{2} \lambda_T^3 \right)^m (\rho_n^m - \rho_p^m) \right\}$$

*H.R. Jaqaman, A.Z. Mekjian and L. Zamick, PRC (1983) 2782.*

c) **Transport models (more realistic approach):**

*Bao-An Li, Phys. Rev. Lett. 88 (2002) 192701, and several papers by others*

# Symmetry energy and single nucleon potential used in the IBUU04 transport model



The  $x$  parameter is introduced to mimic various predictions on the symmetry energy by different microscopic nuclear many-body theories using different effective interactions

Default: Gogny force

Potential energy density

$$V(\rho, \delta) = \frac{A_1}{2\rho_0} \rho^2 + \frac{A_2}{2\rho_0} \rho^2 \delta^2 + \frac{B}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} (1 - x\delta^2) + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \int \int d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}$$

Single nucleon potential within the HF approach using a modified Gogny force:

$$U(\rho, \delta, \bar{p}, \tau, x) = A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_\tau}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^\sigma (1 - x\delta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho_\tau + \frac{2C_{\tau, \tau}}{\rho_0} \int d^3 p' \frac{f_\tau(r, p')}{1 + (p - p')^2 / \Lambda^2} + \frac{2C_{\tau, \tau'}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(r, p')}{1 + (p - p')^2 / \Lambda^2}$$

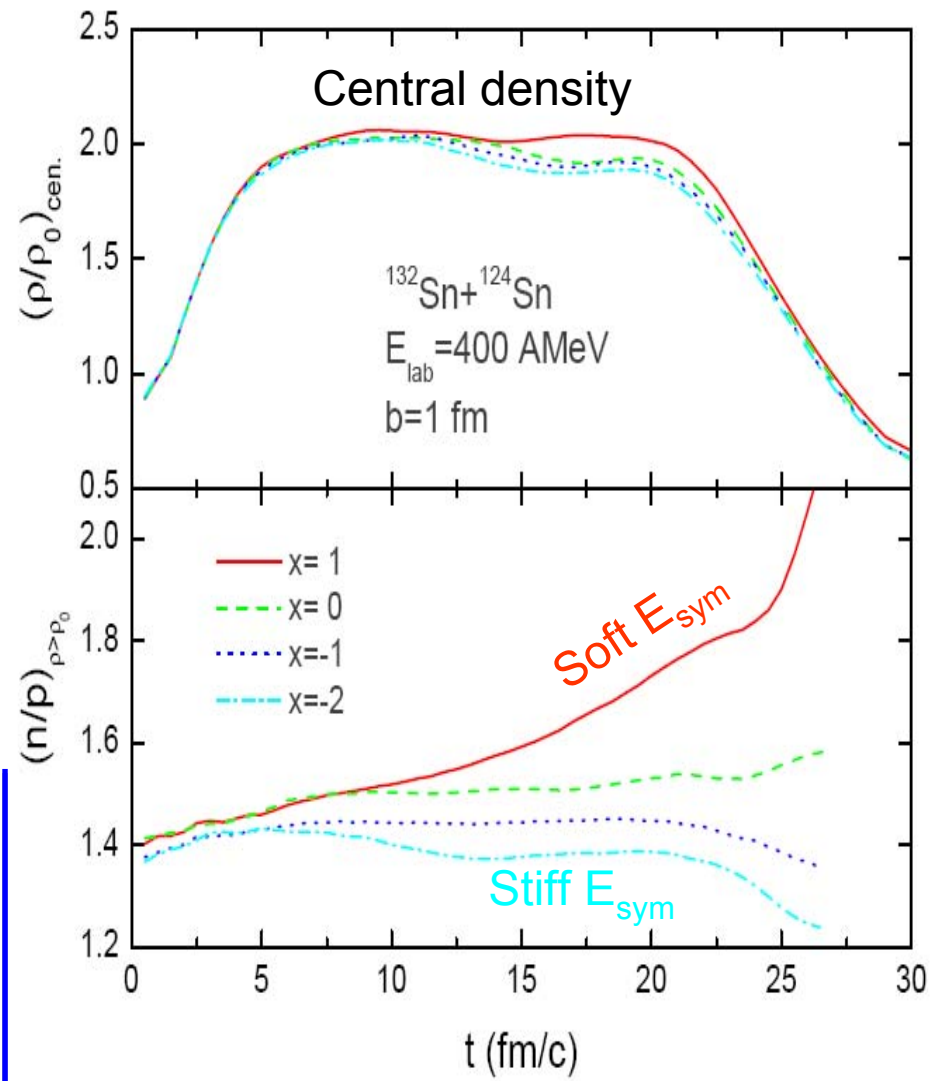
$$\tau, \tau' = \pm \frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma + 1}, A_u(x) = -96 - \frac{2Bx}{\sigma + 1}, K_0 = 211 \text{ MeV}$$

C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

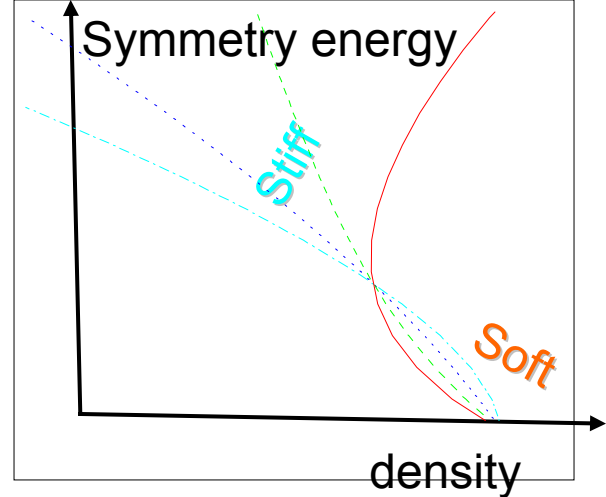
B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC 69, 034614; NPA 735, 563 (2004).

# Formation of dense, asymmetric nuclear matter

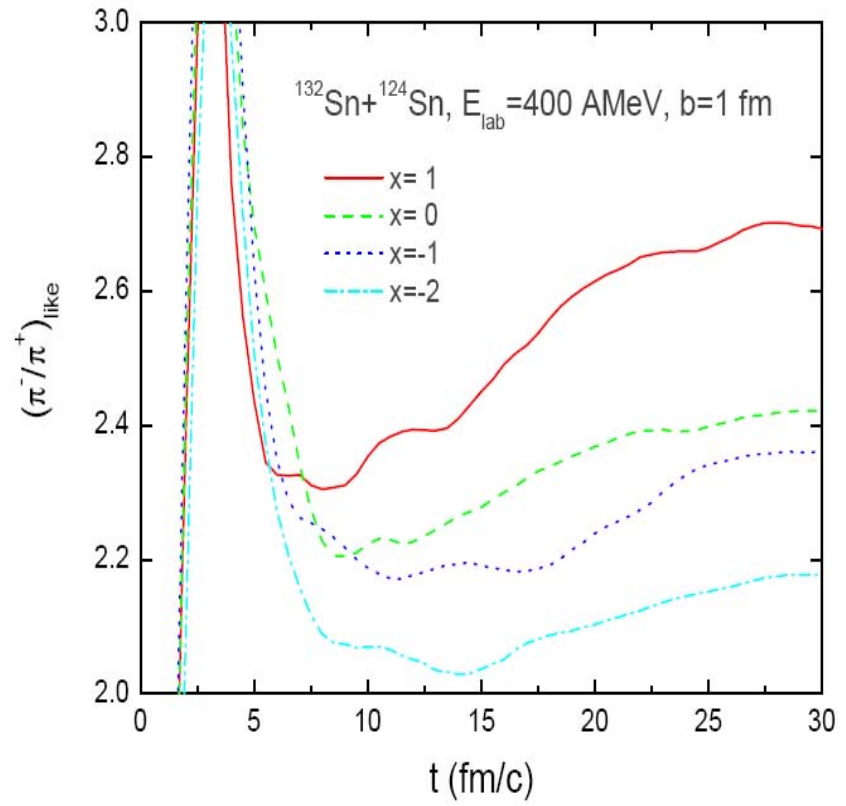
$$E(\rho, \delta) = E(\rho, 0) + E_{sym}(\rho)\delta^2$$



**n/p ratio at supra-normal densities**

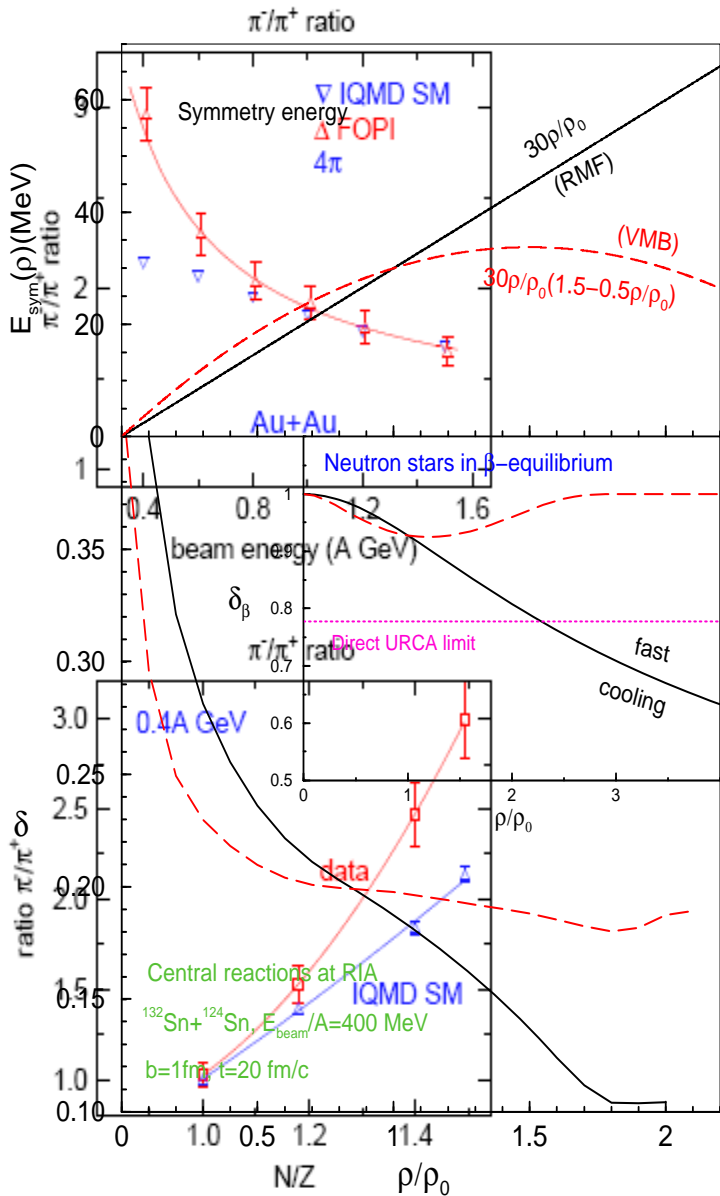


## $\pi^-/\pi^+$ probe of dense matter



# Near-threshold $\pi/\pi^+$ ratio as a probe of symmetry energy at supra-saturation densities

W. Reisdorf et al. for the FOPI collaboration , NPA781 (2007) 459



**IQMD: Isospin-dependent Quantum Molecular Dynamics**  
 C. Hartnack, [Rajeev K. Puri](#), [J. Aichelin](#), [J. Konopka](#),  
[S.A. Bass](#), [H. Stoecker](#), [W. Greiner](#)  
 Eur. Phys. J. A1 (1998) 151-169

$$V_{sym}^{ij} = t_6 \frac{1}{\rho_0} T_{3i} T_{3j} \delta(\vec{r}_i - \vec{r}_j) \quad t_6 = 100 \text{ MeV}$$

corresponding to  $E_{sym}(\rho) = \frac{100}{8} \frac{\rho}{\rho_0} + (2^{2/3} - 1) \frac{3}{5} E_F^0 \left(\frac{\rho}{\rho_0}\right)^{2/3}$

Need a symmetry energy softer than the above to make the pion production region more neutron-rich!

$$E(\rho, \delta) = E(\rho, 0) + E_{sym}(\rho) \delta^2$$

**low (high)** density region is more neutron-rich with **stiff (soft)** symmetry energy

*Bao-An Li, Phys. Rev. Lett. 88 (2002) 192701*

# Circumstantial evidence for a **super-soft** symmetry energy at high densities

Transport model analysis using IBUU04  
 Z. Xiao, B.A. Li, L.W. Chen, G.C. Yong  
 and M. Zhang, PRL 102, 062502 (2009)

