Supplemental Material for "Automated Generation of Arbitrarily Many Kochen-Specker and Other Contextual Sets in Odd Dimensional Hilbert Spaces"

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3-dim

One of eight 51-37 MMPH*s generated by* $\{0, \pm 1, \pm 2, 5\}$*, nonisomorphic to Conway-Kochen's* MMPH

51-37 `213,35b,bYh,hgR,RQN,N6C,CBD,DaF,FEG,GHI,IOP,PTU,UpK,KJL,LV8,89A,Akl,lcf,fde,eZX,XWn,nm2,456,783,MNL,STR,WDA,XYK,ZaV,ZI6,cG2,cYC,WT5,cVT,ijP,oSF,jba.` `1`=(-1,5,2); `2`=(2,0,1); `3`=(1,1,-2); `4`=(1,1,2); `5`=(1,-1,0); `6`=(1,1,-1); `7`=(5,-1,2); `8`=(0,2,1); `9`=(5,1,-2); `A`=(1,-1,2); `B`=(1,2,-1); `C`=(1,0,1); `D`=(-1,1,1); `E`=(-2,5,-1); `F`=(2,1,1); `G`=(1,0,-2); `H`=(2,5,1); `I`=(2,-1,1); `J`=(5,-2,-1); `K`=(1,2,1); `L`=(0,1,-2); `M`=(5,2,1); `N`=(-1,2,1); `O`=(-2,1,5); `P`=(1,2,0); `Q`=(1,-2,5); `R`=(2,1,0); `S`=(1,-2,0); `T`=(0,0,1); `U`=(2,-1,0); `V`=(1,0,0); `W`=(1,1,0); `X`=(1,-1,1); `Y`=(1,0,-1); `Z`=(0,1,1); `a`=(0,1,-1); `b`=(1,1,1); `c`=(0,1,0); `d`=(-2,5,1); `e`=(2,1,-1); `f`=(1,0,2); `g`=(-1,2,5); `h`=(1,-2,1); `i`=(2,-1,5); `j`=(-2,1,1); `k`=(1,5,2); `l`=(2,0,-1); `m`=(1,5,-2); `n`=(-1,1,2); `o`=(-2,-1,5); `p`=(-1,-2,5).

Chosen MMPHs

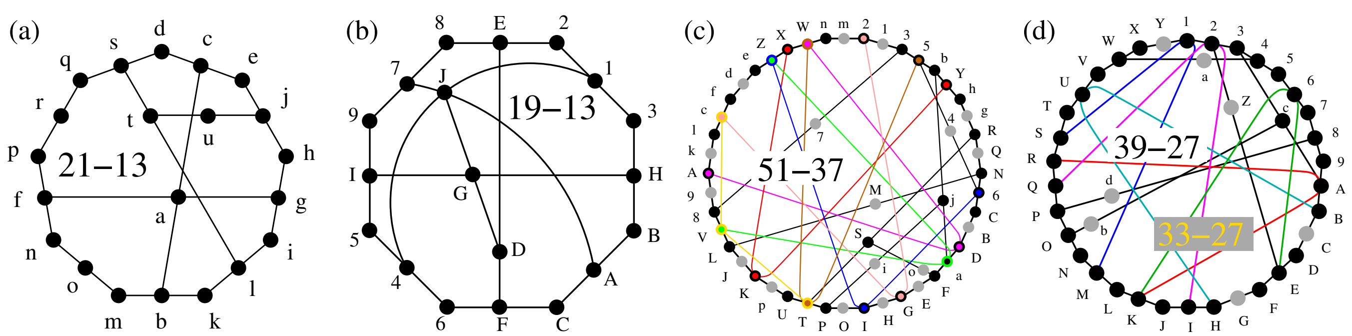

FIG. 1. (a) BMMPH obtained by Voráček and Navara [1]; the 18-13 MMPH which one obtains by removing the vertices `o,r,u` is a NBMMPH; (b) KS MMPH which does not have a real coordinatization and most probably also not a complex one; (c) One of nine 51-37 KS MMPHs we generated; it is a 22-gon non-isomorphic to Conway-Kochen's 51-37; (d) KS NBMMPH 39-27 does not have a known coordinatization—non-KS NBMMPH 33-27 obtained by removal of all gray vertices from the 39-27 therefore also does not have a known coordinatization.

39-27 critical KS MMPH*—possible coordinatization is an open problem*

39-27 1st 17 triads build a 17-gon: `837,7NF,FEG,GLK,K4c,cbY,Y2S,SRT,TWX,X6H,HJI,IQV,Vd5,5UA,A9B,BDC,CZ8,123,456,MND,OPQ,UJ8,aVL,E62,dW3,P74,cW9.`

Other MMPH*s generated by* $\{0,\pm1,\pm2,5\}$

53-38 213,39A,AFG,GpB,BNX,XWY,YdK,KVf,fe5,546,678,8ED,DIr,rqO,OLP,Pkl,lCH,HMa,aZb,bhJ,JSj,ji2,BC5, HI2,JEC,KIG,L63,MNL,LKJ,QRS,TUV,cd8,ghA,mnP,opO,nFE,UND,RMF. 1=(5,2,1); 2=(-1,2,1); 3=(0,1,-2); 4=(5,-1,2); 5=(1,1,-2); 6=(0,2,1); 7=(5,1,-2); 8=(1,-1,2); 9=(5,-2,-1); A=(1,2,1); B=(1,1,1); C=(1,-1,0); D=(-1,1,1); E=(1,1,0); F=(1,-1,1); G=(1,0,-1); H=(1,1,-1); I=(1,0,1); J=(0,0,1); K=(0,1,0); L=(1,0,0); M=(0,1,1); N=(0,1,-1); O=(0,1,2); P=(0,2,-1); Q=(2,1,5); R=(2,1,-1); S=(1,-2,0); T=(-2,5,-1); U=(2,1,1); V=(1,0,-2); W=(2,5,-1); X=(-2,1,1); Y=(1,0,2); Z=(-2,1,5); a=(2,-1,1); b=(1,2,0); c=(1,5,2); d=(2,0,-1); e=(-1,5,2); f=(2,0,1); g=(-1,-2,5); h=(2,-1,0); i=(1,-2,5); j=(2,1,0); k=(5,-1,-2); l=(1,1,2); m=(5,1,2); n=(-1,1,2); o=(5,2,-1); p=(1,-2,1); q=(5,-2,1); r=(1,2,-1).

One of the eight 54-39 MMPHs:

54-39 546,6DE,EmW,WRV,VUJ,JHI,Ipq,qTs,srG,GFC,CAB,B38,879,9ZL,LMN,NOP,PbY,Yci,ihj,jdg,gef,fXa,a25, 123,KLJ,QRP,STN,XYG,bI3,cE9,cT2,dC6,dbZ,XV8,dVT,klR,nSB,oU5,laZ. 1=(-1,1,2); 2=(1,1,0); 3=(1,-1,1); 4=(5,1,-2); 5=(1,-1,2); 6=(0,2,1); 7=(1,-2,1); 8=(1,0,-1); 9=(1,1,1); A=(5,-2,-1); B=(1,2,1); C=(0,1,-2); D=(5,-1,2); E=(1,1,-2); F=(5,2,1); G=(-1,2,1); H=(-2,5,1); I=(2,1,-1); J=(1,0,2); K=(2,5,-1); L=(-2,1,1); M=(2,-1,5); N=(1,2,0); O=(-2,1,5); P=(2,-1,1); Q=(2,5,1); R=(1,0,-2); S=(2,-1,0); T=(0,0,1); U=(2,0,-1); V=(0,1,0); W=(2,0,1); X=(1,0,1); Y=(1,1,-1); Z=(0,1,-1); a=(-1,1,1); b=(0,1,1); c=(1,-1,0); d=(1,0,0); e=(5,-2,1); f=(1,2,-1); g=(0,1,2); h=(5,-1,-2); i=(1,1,2); j=(0,2,-1); k=(-2,5,-1); l=(2,1,1); m=(-1,5,2); n=(-1,-2,5); o=(1,5,2); p=(2,1,5); q=(1,-2,0); r=(1,-2,5); s=(2,1,0).

55-40 123,456,789,AB6,CD9,EF3,GHB,IJD,KLF,MN9,OP6,QR3,SNH,TRJ,UPL,VW5,XY8,Za2,bcB,dcD,ecF,eaN,dYP, bWR,RPN,fgS,hiU,jkT,lmH,noJ,pqL,rM5,sQ8,tO2,mXW,qaV,oZY,gdV,keX,ibZ. 1=(5,-1,2), 2=(1,1,-2), 3=(0,2,1), 4=(2,5,1), 5=(2,-1,1), 6=(1,0,-2), 7=(-1,-2,5), 8=(1,2,1), 9=(2,-1,0), A=(-2,5,-1), B=(2,1,1), C=(1,2,5), D=(1,2,-1), E=(5,1,-2), F=(1,-1,2), G=(-2,-1,5), H=(1,-2,0), I=(5,-2,1), J=(0,1,2), K=(1,5,2), L=(2,0,-1), M=(1,2,0), N=(0,0,1), O=(2,0,1), P=(0,1,0), Q=(0,1,-2), R=(1,0,0), S=(2,1,0), T=(0,2,-1), U=(1,0,2), V=(1,1,-1), W=(0,1,1), X=(1,-1,1), Y=(1,0,-1), Z=(1,1,1), a=(1,-1,0), b=(0,1,-1), c=(-1,1,1), d=(1,0,1), e=(1,1,0), f=(1,-2,5), g=(-1,2,1), h=(2,5,-1), i=(-2,1,1), j=(5,1,2), k=(-1,1,2), l=(2,1,5), m=(2,1,-1), n=(5,2,-1), o=(1,-2,1), p=(-1,5,-2), q=(1,1,2), r=(-2,1,5), s=(5,-2,-1), t=(-1,5,2)

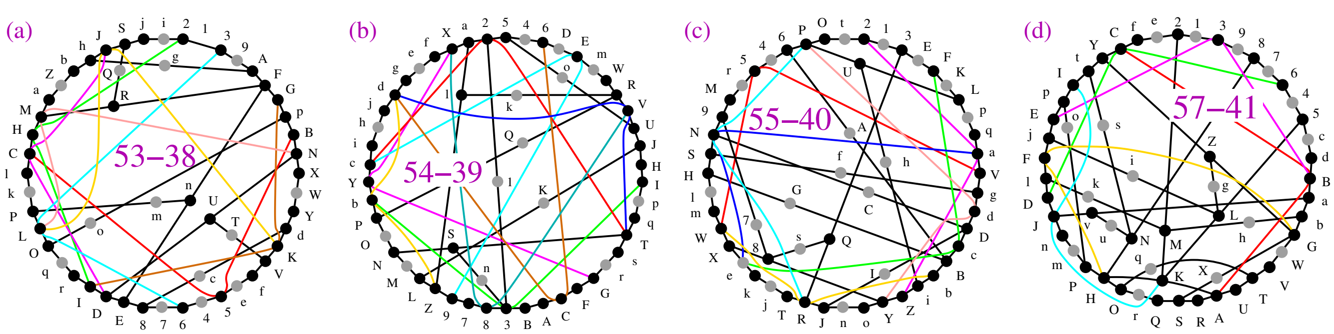

FIG. 2. (a-c) Critical 3D MMPHs generated by the components $\{0,\pm1,\pm2,5\}$; (a) the only 53-38; 22-gon; (b) one of the eight 54-39s; 23-gon; (c) the only 55-40; 22-gon; (d) the only 57-41—the smallest MMPH generated by $\{0,\pm1,2,\pm5,\pm\omega,2\omega\}$; 21-gon.

The smallest MMPH *generated by* $\{0,\pm1,2,\pm5,\pm\omega,2\omega\}$*, shown in Fig. (d)*

57-41 213,398,876,645,5cd,dBa,abG,GWV,VTU,UAR,RSQ,QrO,OHP,Pmn,nJD,DlF,FjE,EpI,ItY,YCf,fe2,ABC,DC6, EB3,FGH,IJK,LK5,MK2,NH8,KHA,XSG,YZG,gZL,hbM,ijL,klM,opP,qTO,stN,uvN,vaJ. 1=(1,2,5); 2=(2,-1,0); 3=(1,2,-1); 4=(-1,2,5); 5=(2,1,0); 6=(-1,2,-1); 7=(5,2,-1); 8=(0,1,2); 9=(-5,2,-1); A=(0,1,0); B=(1,0,1); C=(1,0,-1); D=(1,1,1); E=(-1,1,1); F=(0,1,-1); G=(0,1,1); H=(1,0,0); I=(1,1,0); J=(1,-1,0); K=(0,0,1); L=(-1,2,0); M=(1,2,0); N=(0,2,-1); O=(0,-1,2); P=(0,2,1); Q=(-1,2ω,ω); R=(1,0,ω); S=(1,ω,-ω); T=(1,2ω,ω); U=(1,0,-ω); V=(1,-ω,ω); W=(2,ω,-ω); X=(2,-ω,ω); Y=(1,-1,1); Z=(2,1,-1); a=(1,1,-1); b=(2,-1,1); c=(-1,2,-5); d=(-1,2,1); e=(1,2,-5); f=(1,2,1); g=(2,1,5); h=(2,-1,-5); i=(2,1,-5); j=(2,1,1); k=(2,-1,5); l=(2,-1,-1); m=(5,-1,2); n=(-1,-1,2); o=(-5,-1,2); p=(1,-1,2); q=(-5,2ω,ω); r=(5,2ω,ω); s=(5,1,2); t=(-1,1,2); u=(-5,1,2); v=(1,1,2).

Original KS *set*

The original 192-118 KS set found by Kochen and Specker [2] was not accompanied by an explicit coordinatization. Instead, they give the equation $\sin(\pi/10) = xy/\sqrt{(1+x^2+x^2y^2)(1+y^2+x^2y^2)}$ where x and y, provided they satisfy this equation, are presumably arbitrary parameters that are used to define the vectors in their starting subgraph called Γ_1. The final hypergraph (called Γ_2), shown in Fig. 3, consists of 15 copies of Γ_1, with assumed—but so far undefined—vectors rotated (using rotation matrices) so that the copies align correctly. Notice that Kochen and Specker in their original graphical representation of Γ_2 dropped all 75 gray vertices thus arriving and 117-118 hypergraph. But as we explained in [4, Sec. XII] if one dropped all gray vertices, the remaining hypergraph would not be a KS set any more.

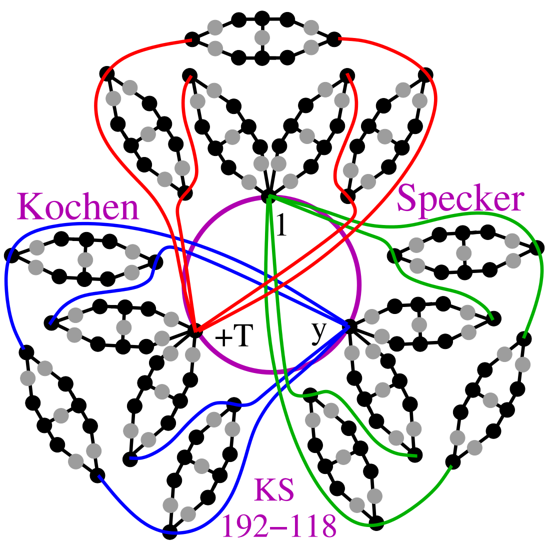

FIG. 3. KS MMPH of the original Kochen-Specker 192-118 set presented according to [3, Fig. 6]; note that the figure in [3, Fig. 6] and the one-to-one correspondence to [2, Γ_2] presented there is obtained exclusively from the ASCII string of the MMPH without any reference to its coordinatization; note also that one can use the string below to assign its vertices to the vertices of the figure by hand.

The x, y satisfying the equation is not sufficient to provide a coordinatization. E.g., one solution is $x{=}y{=}\sqrt{\varphi}$, where $\varphi{=}(1+\sqrt{5})/2$ is the golden ratio. When we compute vectors for just the 117 black vertices on hypergraph Γ_2, only 115 of them are unique, so we cannot use it even for the truncated non-critical [5] 117-118 set. The complete 192-118 KS MMPH requires $192 - 117 = 75$ additional unique vectors (each computed as the cross product of the other two vectors on the hyperedge), placing an even more severe constraint on the possible values of x and y.

We found that $x{=}1/\varphi$ and $y{=}\sqrt{\varphi}$ yield 192 unique vectors, and we used these values to compute the coordinatization. To show the final result, let us define $f(p,q,r) = \sqrt{2^p\sqrt{5}^q\varphi^r}$. Then we define 16 constants as follows: $c_1{=}f(\text{-}1,0,0)$; $c_2{=}f(0,0,\text{-}1)$; $c_3{=}f(\text{-}1,1,\text{-}3)$; $c_4{=}f(\text{-}1,0,\text{-}2)$; $c_5{=}f(0,\text{-}1,\text{-}6)$; $c_6{=}f(\text{-}1,\text{-}1,\text{-}5)$; $c_7{=}f(2,\text{-}1,0)$; $c_8{=}f(\text{-}1,\text{-}1,\text{-}3)$; $c_9{=}f(0,\text{-}1,\text{-}2)$; $c_{10}{=}f(0,0,\text{-}5)$; $c_{11}{=}f(\text{-}1,0,\text{-}6)$; $c_{12}{=}f(2,\text{-}1,\text{-}2)$; $c_{13}{=}f(\text{-}1,\text{-}1,3)$; $c_{14}{=}f(2,\text{-}1,\text{-}4)$; $c_{15}{=}f(\ln(9/2-9/\sqrt{5})/\ln 2,0,0){=}3\sqrt{(5-2\sqrt{5})/10}$; $c_{16}{=}f(\ln(17/4{-}31/(4\sqrt{5}))/\ln 2,0,0){=}\frac{1}{2}\sqrt{(85-31\sqrt{5})/5}$.

When normalized, the 192 vectors have 73 different components. By using unnormalized vectors, i.e. rays, we reduced the number of different components to 24, whose values are $0, 1, \pm c_1, \pm c_2, c_3, c_4, -c_5, \pm c_6, c_7, \pm c_8, \pm c_9, -c_{10}, \pm c_{11}, -c_{12}, -c_{13}, -c_{14}, -c_{15}$, and $-c_{16}$. The coordinatization is presented below.

192-118
```
123,345,567,789,9AB,BC1,DEF,FGH,HIJ,JKL,LMN,NOD,PQR,RST,TUV,VWX,XYZ,ZaP,bcd,def,fgh,
hij,jkl,lmb,1no,opq,qrs,stu,uvw,wx1,yz!,!"#,#$%,%&',’(),)*y,-/:,:;<,<=>,>?@,@[\,\]-,^_‘,‘{|,|}~,
~+1+2,+2+3+4,+4+5^,+6+7+8,+8+9+A,+A+B+C,+C+D+E,+E+F+G,+G+H+6,y+I+J,+J+K+L,+L+M+N,+N+O+P,+P+Q+R,
+R+Sy,+T+U+V,+V+W+X,+X+Y+Z,+Z+a+b,+b+c+d,+d+e+T,+f+g+h,+h+i+j,+j+k+l,+l+m+n,+n+o+p,+p+q+f,+r+s+t,
+t+u+v,+v+w+x,+x+y+z,+z+!+",+"+#+r,+$+%+&,+&+’+(,+(+)+*,+*+-+/,+/+:+;,+;+<+$,+T+=+>,+>+?+@,
+@+[+\,+\+]+^,+^+_+‘,+‘+{+T,4A+|,GM+},SY+~,ek++1,pv++2,"(++3,;[++4,{+3++5,+9+F++6,+K+Q++7,
+W+c++8,+i+o++9,+u+!++A,+’+:++B,+?+_++C,D7y,PJy,bVy,shy,-%+T,^>+T,~+C+T,+N+6+T,+f+Z1,+r+l1,
+$+x1,+\+*1,1y+T.
```

1=(1,0,0); 2=(0,-c_2,c_4); 3=(0,c_4,c_2); 4=(c_8,-c_{10},c_{11}); 5=(c_1,c_9,-c_8); 6=(-c_1,c_7,c_8); 7=(c_8,0,c_1); 8=(-c_1,-c_{12},c_8); 9=(c_4,-c_9,-c_6); A=(c_4,c_9,c_6); B=(0,c_6,-c_9); C=(0,c_9,c_6); D=(c_1,0,-c_8); E=(c_8,-c_{12},c_1); F=(c_6,c_9,c_4); G=(c_1,-c_2,c_3); H=(-c_{13},-c_9,c_1); I=(c_8,-c_{12},-c_{11}); J=(c_3,0,c_1); K=(-c_8,-c_{14},c_{11}); L=(c_8,-c_5,-c_{11}); M=(c_1,c_2,c_8); N=(c_8,-c_9,c_1); O=(c_8,c_7,c_1); P=(-c_{13},0,c_1); Q=(-c_{11},-c_{12},-c_8); R=(-c_1,c_9,-c_{13}); S=(c_4,c_9,-c_6); T=(c_3,-c_2,-c_1); U=(c_1,c_7,-c_{13}); V=(c_1,0,c_3); W=(c_{11},-c_{12},-c_8); X=(c_1,c_9,-c_{13}); Y=(c_4,-c_9,c_6); Z=(c_3,c_2,c_1); a=(-c_1,c_7,-c_{13}); b=(c_1,0,-c_{13}); c=(c_8,-c_{14},c_{11}); d=(-c_8,-c_5,-c_{11}); e=(c_1,-c_2,-c_8); f=(-c_8,-c_9,c_1); g=(-c_8,c_7,c_1); h=(c_1,0,c_8); i=(-c_8,-c_{12},c_1); j=(-c_6,c_9,c_4); k=(-c_1,-c_2,c_3); l=(-c_{13},c_9,-c_1); m=(-c_8,-c_{12},-c_{11}); n=(0,c_2,c_4); o=(0,c_4,-c_2); p=(c_8,-c_{10},-c_{11}); q=(c_1,c_9,c_8); r=(-c_1,c_7,-c_8); s=(-c_8,0,c_1);

t=(c_4,c_7,c_6); u=$(c_4,-c_5,c_6)$; v=$(c_3,c_2,-c_1)$; w=$(0,c_1,c_2)$; x=$(0,-c_2,c_1)$; y=(0,1,0); z=$(c_6,0,c_9)$; !=$(c_9,0,-c_6)$; "=(c_6,c_4,c_9); #=$(-c_6,c_4,-c_9)$; $=$(c_8,-c_1,-c_{12})$; %=$(c_1,c_8,0)$; &=$(c_8,-c_1,c_7)$; ’=$(-c_8,c_1,c_9)$; (=$(c_{11},c_8,-c_{10})$;)=$(c_2,0,c_4)$; *=$(c_4,0,-c_2)$; -=$(-c_8,c_1,0)$; /=$(c_1,c_8,-c_{12})$; :=$(c_4,c_6,c_9)$; ;=$(c_3,c_1,-c_2)$; <=$(c_1,-c_{13},-c_9)$; ==$(-c_{11},c_8,-c_{12})$; >=$(c_1,c_3,0)$; ?=$(c_{11},-c_8,-c_{14})$; @=$(-c_{11},c_8,-c_5)$; [=$(c_8,c_1,c_2)$; \=$(c_1,c_8,-c_9)$;]=$(c_1, c_8,c_7)$; ^=$(c_1,-c_{13},0)$; _=$(c_8,c_{11},-c_{14})$; ‘=$(-c_8,-c_{11},-c_5)$; {=$(0,-c_9,c_4)$; |=$(c_8,-c_{11},-c_5)$; }=$(-c_8,c_{11},-c_{14})$; ~=$(c_3,c_1,0)$; +1=$(-c_{13},c_1,c_7)$; +2=$(-c_1,c_3,-c_2)$; +3=$(-c_6,c_4,c_9)$; +4=$(-c_{13},-c_1,c_9)$; +5=$(-c_8,-c_{11},-c_{12})$; +6=$(c_8,c_1,0)$; +7=$(c_1,-c_8,-c_{12})$; +8=$(c_4,-c_6,c_9)$; +9=$(c_3,-c_1,-c_2)$; +A=$(-c_1,-c_{13},c_9)$; +B=$(-c_{11},-c_8,-c_{12})$; +C=$(-c_{13},c_1,0)$; +D=$(c_{11},c_8,-c_{14})$; +E=$(-c_{11},-c_8,-c_5)$; +F=$(-c_8,c_1,-c_2)$; +G=$(c_1,-c_8,-c_9)$; +H=$(c_1,-c_8,c_7)$; +I=$(c_1,0,-c_2)$; +J=$(c_2,0,c_1)$; +K=$(-c_1,c_3,c_2)$; +L=$(c_6,c_4,-c_5)$; +M=$(c_6,c_4,c_7)$; +N=$(c_1,-c_8,0)$; +O=$(-c_8,-c_1,c_7)$; +P=$(c_8,c_1,c_9)$; +Q=$(-c_{11},c_8,-c_{10})$; +R=$(-c_2,0,c_4)$; +S=$(c_4,0,c_2)$; +T=(0,0,1); +U=$(-c_2,c_1,0)$; +V=$(c_1,c_2,0)$; +W=$(c_2,-c_1,c_3)$; +X=$(-c_5,c_6,c_4)$; +Y=$(c_7,c_6,c_4)$; +Z=$(0,c_1,-c_8)$; +a=$(c_7,-c_8,-c_1)$; +b=$(c_9,c_8,c_1)$; +c=$(-c_{10},-c_{11},c_8)$; +d=$(c_4,-c_2,0)$; +e=$(c_2,c_4,0)$; +f=$(0,c_8,c_1)$; +g=$(-c_{12},c_1,-c_8)$; +h=$(c_9,c_4, -c_6)$; +i=$(-c_2,c_3,-c_1)$; +j=$(c_9,-c_1,-c_{13})$; +k=$(-c_{12},-c_{11},-c_8)$; +l=$(0,-c_{13},c_1)$; +m=$(-c_{14}, c_{11},c_8)$; +n=$(-c_5,-c_{11},-c_8)$; +o=$(-c_2,-c_8,c_1)$; +p=$(-c_9,c_1,-c_8)$; +q=$(c_7,c_1,-c_8)$; +r=$(0,c_3,c_1)$; +s=$(c_7,-c_{13},c_1)$; +t=$(-c_2,-c_1,c_3)$; +u=$(c_9,-c_6,c_4)$; +v=$(c_9,-c_{13},-c_1)$; +w=$(-c_{12},-c_8,-c_{11})$; +x=$(0,c_1,-c_{13})$; +y=$(-c_{14},c_8,c_{11})$; +z=$(-c_5,-c_8,-c_{11})$; +!=$(c_4,0,-c_9)$; +"=$(-c_5,c_8, -c_{11})$; +#=$(-c_{14},-c_8,c_{11})$; +$=$(0,c_1,c_3)$; +%=$(-c_{12},-c_{11},c_8)$; +&=$(-c_9,c_1,-c_{13})$; +’=$(-c_2,c_3,c_1)$; +(=(c_9,c_4,c_6); +)=$(-c_{12},c_1,c_8)$; +*=$(0,-c_8,c_1)$; +-=(c_7,c_1,c_8); +/=$(-c_9,c_1,c_8)$; +:=(c_2,c_8,c_1); +;=$(-c_5,-c_{11},c_8)$; +<=$(-c_{14},c_{11},-c_8)$; +==$(c_9,c_6,0)$; +>=$(c_6,-c_9,0)$; +?=(c_9,c_6,c_4); +@=$(-c_9,-c_6,c_4)$; +[=$(-c_{12},c_8,-c_1)$; +\=$(0,c_1,c_8)$; +]=$(c_7,c_8,-c_1)$; +^=$(c_9,-c_8,c_1)$; +_=$(-c_{10},c_{11},c_8)$; +‘=$(c_4,c_2,0)$; +{=$(-c_2,c_4,0)$; +|=$(c_4,-c_5,-c_{16})$; +}=$(-c_9,c_6,c_2)$; +~=$(0,c_6,c_9)$; ++1=(c_9,c_6,c_2); ++2=$(-c_1,-c_5,-c_{15})$; ++3=$(-c_{16},c_4,-c_5)$; ++4=$(c_2,-c_9,c_6)$; ++5=$(-c_{13},-c_{11},-c_5)$; ++6=(c_2,c_9,c_6); ++7=$(-c_{15},-c_1,-c_5)$; ++8=$(-c_5,-c_{15}, -c_1)$; ++9=(c_6,c_2,c_9); ++A=$(-c_5,-c_{13},-c_{11})$; ++B=$(c_6,c_2,-c_9)$; ++C=$(-c_5,-c_{16},c_4)$.

The components may be used to generate a master MMPH whose class contains the MMPH 192-118. The master has 2416 vertices and 1432 hyperedges.

5-dim

29-16 HOINJ,JGSTF,FT4Q8,85679,92LOH,PQRST,KLMNO,CDEIO,ABEGT,34DMO,12BRT,237CO,146AT,279OP, 468KT,5EJOT. 1=(1,-1,1,0,-1); 2=(1,0,-1,0,0); 3=(1,-1,1,1,0); 4=(0,1,1,0,0); 5=(0,0,1,0,0); 6=(1,0,0,0,1); 7=(0,1,0,1,0); 8=(1,0,0,0,-1); 9=(0,1,0,-1,0); A=(1,1,-1,0,-1); B=(1,1,1,0,1); C=(1,1,1,-1,0); D=(1,1,-1,1,0); E=(1,-1,0,0,0); F=(1,-1,1,0,1); G=(0,0,1,0,-1); H=(1,-1,1,-1,0); I=(0,0,1,1,0); J=(1,1,0,0,0); K=(0,1,-1,0,0); L=(1,1,1,1,0); M=(1,0,0,-1,0); N=(1,-1,-1,1,0); O=(0,0,0,0,1); P=(1,0,1,0,0); Q=(1,1,-1,0,1); R=(0,1,0,0,-1); S=(-1,1,1,0,1); T=(0,0,0,1,0).

31-17 12345,16789,16ABC,13DEB,14D8F,1GHF9,1IJ8K,1IHAE,LMGI5,L27NO,L2PQC,M27RS,2TJUO,2THPR,23VUN, 24VPS,2PQRS. 1=(0,0,0,0,1); 2=(0,0,0,1,0); 3=(1,1,0,0,0); 4=(1,-1,0,0,0); 5=(0,0,1,0,0); 6=(1,0,0,-1,0); 7=(0,1,1,0,0); 8=(1,1,-1,1,0); 9=(1,-1,1,1,0); A=(1,1,1,1,0); B=(1,-1,-1,1,0); C=(0,1,-1,0,0); D=(0,0,1,1,0); E=(1,-1,1,-1,0); F=(1,1,1,-1,0); G=(0,1,0,1,0); H=(1,0,-1,0,0); I=(0,1,0,-1,0); J=(1,0,1,0,0); K=(-1,1,1,1,0); L=(1,0,0,0,1); M=(1,0,0,0,-1); N=(1,-1,1,0,-1); O=(1,1,-1,0,-1); P=(1,1,1,0,-1); Q=(-1,1,1,0,1); R=(1,-1,1,0,1); S=(1,1,-1,0,1); T=(0,1,0,0,1); U=(1,-1,-1,0,1); V=(0,0,1,0,1).

7-dim

13-4 1234567,456789A,123ABCD,4567BCD.

34-14 1234567, 189A5BC, 189DE7F, 189GHIJ, 189KHBL, 2MNDOIP, 2MNEOCL, 2MNGK6F, QRNSAJP, QT4U567, RTV9567, WXMS567, WYV8AJP, XY3U567. 1=(0,0,0,1,0,0,0); 2=(0,0,1,0,0,0,0); 3=(1,-1,0,0,0,0,0); 4=(1,1,0,0,0,0,0); 5=(0,0,0,0,0,0,1); 6=(0,0,0,0,1,1,0); 7=(0,0,0,0,1,-1,0); 8=(0,1,-1,0,0,0,0); 9=(0,1,1,0,0,0,0); A=(0,0,0,0,1,0,0); B=(1,0,0,0,0,-1,0); C=(1,0,0,0,0,1,0); D=(1,0,0,0,1,1,-1); E=(-1,0,0,0,1,1,1); F=(1,0,0,0,0,0,1); G=(1,0,0,0,1,-1,-1); H=(1,0,0,0,1,1,1); I=(1,0,0,0,-1,0,0); J=(0,0,0,0,0,1,-1); K=(1,0,0,0,-1,1,-1); L=(0,0,0,0,1,0,-1); M=(0,1,0,1,0,0,0); N=(0,1,0,-1,0,0,0); O=(1,0,0,0,1,-1,1); P=(0,0,0,0,0,1,1); Q=(-1,1,1,1,0,0,0); R=(1,1,-1,1,0,0,0); S=(1,0,1,0,0,0,0); T=(1,-1,1,1,0,0,0); U=(0,0,1,-1,0,0,0); V=(1,0,0,-1,0,0,0); W=(1,-1,-1,1,0,0,0); X=(1,1,-1,-1,0,0,0); Y=(1,1,1,1,0,0,0).

28(58)-14 3451678, 4LF8BJK, PHG7QRS, 349A16(T), 38BCD(UV), F8QJO(WX), 4HIJK(YZ), AMNCD(ab), 3452(cde), EFG2(fgh), 9MNO(ijk), PLRS(lmn), EI7(opqr), 12(stuvw). 1=(0,1,0,0,0,0,0); 2=(1,0,0,0,0,0,0); 3=(0,0,1,1,1,-1,0); 4=(0,0,1,1,-1,1,0); 5=(0,0,1,-1,0,0,0); 6=(1,0,0,0,0,0,1); 7=(1,0,0,0,1,1,-1); 8=(-1,0,0,0,1,1,1); 9=(0,0,1,-1,1,1,0); A=(0,0,-1,1,1,1,0); B=(1,0,-1,1,0,0,1); C=(1,-1,0,-1,1,0,0); D=(1,1,1,0,0,1,0); E=(0,1,0,-1,-1,1,0); F=(0,1,1,0,0,-1,1); G=(0,-1,1,0,0,1,1); H=(0,1,0,1,1,0,1); I=(0,1,1,0,0,-1,-1); J=(1,-1,1,0,1,0,0); K=(1,1,0,-1,0,1,0); L=(0,1,0,1,1,0,-1); M=(0,1,0,0,1,-1,-1); N=(0,1,-1,-1,0,0,1); O=(1,1,0,1,0,1,0); Q=(1,0,-1,-1,0,0,1); P=(0,0,1,-1,1,-1,0); R=(1,1,1,0,-1,0,0); S=(1,-1,0,1,0,-1,0); T=(1,0,0,0,0,0,-1); U=(0,1,0,-1,0,-1,1); V=(0,1,-1,0,1,0,-1); W=(0,0,1,-1,-1,1,0); X=(0,1,0,-1,1,0,-1); Y=(1,0,-1,1,0,0,-1); Z=(1,0,0,0,-1,-1,1); a=(1,0,0,1,0,-1,1); b=(-1,0,1,0,1,0,1); c=(0,1,0,0,0,0,1); d=(0,1,0,0,1,1,-1); e=(0,-1,0,0,1,1,1); f=(0,0,1,1,-1,0,-1); g=(0,0,1,-1,1,0,-1); h=(0,1,0,1,1,1,0); i=(1,-1,-1,0,1,0,0); j=(-1,0,0,1,1,0,1); k=(1,0,1,0,0,-1,1); l=(0,1,-1,0,0,-1,1); m=(1,0,0,0,1,1,1); n=(-1,0,1,1,0,0,1); o=(0,0,1,-1,1,0,1); p=(1,-1,1,0,-1,0,0); q=(-1,0,1,1,0,1,0); r=(1,1,0,0,1,0,-1); s=(0,0,0,1,1,1,-1); t=(0,0,0,1,1,-1,1); u=(0,0,0,1,-1,1,1); v=(0,0,0,-1,1,1,1); w=(0,0,1,0,0,0,0).

207-97 1234567, 89ABCDE, 8FGHIJK, 8LMNOPE, QRS6TUV, QRS6WXY, QRSZab7, QRScde7, QRfg6TW, QRfgZc7, Qf6hijk, QglLmn7, Qaopqrs, QetuvOw, Qxyzq!", Q#$V%&’, Q(yuU)*, QLnX-/r, :;6hN<=, :>?@[\], :^_‘{]|, }~^+1M+27, }~+3+4mG7, }+5+6@+7+8+9, }e+6v<+A+B, }(+C+DY{+E, Rf6+F+GC[, RaoN+H+I+J, Rbo+K+7\+L, R+M+NX-+O+P, R+Q+R+S+7+T!, R+U_Y-+8+V, R+U+WGN+X+Y, R+ZMV%+8+V, R9+aU%+A+I, R+b+RX-+A+I, +c+d6v+e+X+H, +c+ftuv+X+P, +cx+g+h%+i+P, +c+Znv+H!P, +c(+Ru+j)+E, +k+f+l+m+n+O+o, +k+p+N+S+q+r+s, +td+l+K<s+B, +tl+u@+v+w+x, +t+y+z+a+F+q+x, +!+5+"+#k’+$, +!b+%z+&+’+V, +!+(+z+)+#+X+x, +*+f+"+#+w’+-, +*+Z+/+#+:+r+s, +*+Z+;+D‘+<+V, +=+1_U+>+?+9, ~+f+@+[+\+]+P, ~b+^h+\&+A+_, ~+Q+/+mC+‘+s, f+5H+{+GC+|, fxyI+}+I+J, f#AI+}+O+E, f(+~‘++1+A++2, f(y+K+7++3P, f++4_+j++5/++6, f+(M+h+>++7+|, ++8+dde++9++A7, ++8e++9+Sq+>+V, ++8e++A+[k+A’, ++8+MA+K+vq++B, ++8+U++C+[j"+s, ++8+U++CN+v+w+Y, ++8+U+2U++7r+L, ++8+Zm++DO"+s, ++8+ZnX+}r+L, ++8+(+zn+[j++B, ++E+f+^++Fi++G*, ++E+pnV++H\++I, ++E(+C++J++K++3+‘, ++E+y++CX++L+$|, ;+5+^h=&+_, ;x+u++J+vq++M, ;#+z+C+#++N++O, +dx+gB+v[+Y, +d(+gU++7+O*, +d(+Rp+7++3+T, +dF+zGv+X+Y, +dLMX++1/|, +dLMV++7+8+|, g+5+{u++P+:*, g+5+{++D+H&’, g+M+Np+vq++Q, g+M+a+h++7’++R, g+Q+gB+v++K++Q, g+Q+R‘+}’++R, glm++D+XD+s, glnI+}\+L, g9+N‘+}+i*, gF+z+2++D+X++Q, gLnN+H++3P.

1=(0,0,0,1,1,1,-1); 2=(0,0,0,1,1,-1,1); 3=(0,0,0,1,-1,1,1); 4=(0,0,0,-1,1,1,1); 5=(0,0,1,0,0,0,0); 8=(0,0,1,1,1,1,0); Q=(0,0,1,1,1,-1,0); :=(0,0,1,1,-1,0,1); }=(0,0,1,1,-1,0,-1); R=(0,0,1,1,-1,1,0); +c=(0,0,1,1,-1,-1,0); S=(0,0,1,-1,0,0,0); +k=(0,0,1,-1,0,1,1); +t=(0,0,1,-1,0,1,-1); +!=(0,0,1,-1,0,-1,1); +*=(0,0,-1,1,0,1,1); +==(0,0,1,-1,1,0,1); ~=(0,0,1,-1,1,0,-1); f=(0,0,1,-1,1,1,0); ++8=(0,0,1,-1,1,-1,0); ++E=(0,0,1,-1,-1,0,1); ;=(0,0,-1,1,1,0,1); +d=(0,0,1,-1,-1,1,0); g=(0,0,-1,1,1,1,0); 6=(0,1,0,0,0,0,0); Z=(0,1,0,0,0,0,1); c=(0,1,0,0,0,0,-1); d=(0,1,0,0,1,1,1); a=(0,1,0,0,1,1,-1); +f=(0,1,0,0,1,-1,1); +5=(0,1,0,0,1,-1,-1); >=(0,1,0,0,-1,1,-1); e=(0,1,0,0,-1,-1,1); b=(0,-1,0,0,1,1,1); x=(0,1,0,1,0,1,-1); +M=(0,1,0,1,0,-1,1); +Q=(0,1,0,1,0,-1,-1); +U=(0,1,0,1,1,0,1); +Z=(0,1,0,1,1,0,-1); ^=(0,1,0,1,1,1,0); +3=(0,1,0,1,1,-1,0); l=(0,1,0,1,-1,0,-1); +p=(0,1,0,1,-1,1,0); 9=(0,1,0,-1,0,1,1); +b=(0,1,0,-1,0,1,-1); #=(0,1,0,-1,0,-1,1); (=(0,-1,0,1,0,1,1); F=(0,1,0,-1,1,0,1); L=(0,1,0,-1,1,0,-1); +y=(0,1,0,-1,1,-1,0); ++4=(0,1,0,-1,-1,0,1); +(=(0,-1,0,1,1,0,1); +1=(0,1,0,-1,-1,1,0); +4=(0,-1,0,1,1,1,0); m=(0,1,1,0,0,1,1); ++C=(0,1,1,0,0,1,-1); M=(0,1,1,0,0,-1,1); _=(0,1,1,0,0,-1,-1); +g=(0,1,1,0,1,0,1); +N=(0,1,1,0,1,0,-1); +u=(0,1,1,0,1,-1,0); +~=(0,1,1,0,-1,0,1); A=(0,1,1,0,-1,0,-1); +/=(0,1,1,0,-1,1,0); ++9=(0,1,1,1,0,0,-1); +@=(0,1,1,1,0,1,0); +%=(0,1,1,1,1,0,0); +l=(0,1,1,1,-1,0,0); +z=(-1,1,1,1,0,0,0); +6=(0,1,1,-1,0,1,0); +W=(1,-1,-1,1,0,0,0); G=(0,1,-1,0,0,1,-1); n=(0,1,-1,0,0,-1,1); +2=(0,-1,1,0,0,1,1); y=(0,1,-1,0,1,0,1); $=(0,1,-1,0,1,0,-1); +)=(0,1,-1,0,1,-1,0); +R=(0,1,-1,0,-1,0,1); +a=(0,-1,1,0,1,0,1); +C=(0,1,-1,0,-1,1,0); +;=(0,-1,1,0,1,1,0); o=(0,1,-1,1,0,0,1); t=(0,1,-1,1,0,0,-1); ?=(0,1,-1,1,0,-1,0); H=(1,-1,-1,0,1,0,0); +{=(0,1,-1,-1,0,0,1); ++A=(0,-1,1,1,0,0,1); +^=(0,1,-1,-1,0,1,0); u=(-1,1,1,0,0,1,0); +"=(0,-1,1,1,1,0,0); +D=(-1,1,1,0,0,0,1); ++F=(1,-1,-1,0,0,0,1); 7=(1,0,0,0,0,0,0); T=(1,0,0,0,0,0,1); W=(1,0,0,0,0,0,-1); X=(1,0,0,0,1,1,1); U=(1,0,0,0,1,1,-1); ‘=(1,0,0,0,1,-1,1); +h=(1,0,0,0,1,-1,-1); I=(1,0,0,0,-1,1,1); +j=(1,0,0,0,-1,1,-1); Y=(1,0,0,0,-1,-1,1); V=(-1,0,0,0,1,1,1); v=(1,0,0,1,0,1,1); h=(1,0,0,1,0,1,-1); ++D=(1,0,0,1,0,-1,1); +[=(1,0,0,1,0,-1,-1); +S=(1,0,0,1,1,0,1); +F=(1,0,0,1,1,0,-1); +m=(1,0,0,1,1,1,0); z=(1,0,0,1,-1,0,1); +#=(1,0,0,1,-1,-1,0); N=(1,0,0,-1,0,1,1); ++P=(1,0,0,-1,0,1,-1); +e=(1,0,0,-1,0,-1,1); i=(-1,0,0,1,0,1,1); p=(1,0,0,-1,1,0,1); B=(1,0,0,-1,1,0,-1); ++J=(1,0,0,-1,1,1,0); +K=(1,0,0,-1,-1,0,1); +G=(-1,0,0,1,1,0,1); @=(-1,0,0,1,1,1,0); ++N=(1,0,1,0,0,1,1); C=(1,0,1,0,0,-1,1); +:=(1,0,1,0,1,0,1); +X=(1,0,1,0,1,0,-1); ++G=(1,0,1,0,1,1,0); <=(1,0,1,0,1,-1,0); j=(1,0,1,0,-1,0,1); O=(1,0,1,0,-1,0,-1); +q=(1,0,1,0,-1,-1,0); ++5=(1,0,1,1,0,0,1); ++1=(1,0,1,1,0,0,-1); ++L=(1,0,1,1,0,-1,0); +<=(1,0,1,1,-1,0,0);)=(1,0,1,-1,0,0,1); J=(1,0,1,-1,0,0,-1); {=(1,0,1,-1,0,1,0); +v=(1,-1,1,1,0,0,0); [=(1,0,-1,0,0,1,1); +7=(1,0,-1,0,0,1,-1); ++K=(1,0,-1,0,0,-1,1); q=(-1,0,1,0,0,1,1); k=(1,0,-1,0,1,0,1); +w=(1,0,-1,0,1,0,-1); +\=(1,0,-1,0,1,1,0); +&=(1,0,-1,0,1,-1,0); +n=(1,0,-1,0,-1,0,1); +H=(-1,0,1,0,1,0,1); ==(-1,0,1,0,1,1,0); %=(1,0,-1,1,0,0,1); -=(1,0,-1,1,0,0,-1); +>=(1,-1,1,0,-1,0,0); ++7=(1,0,-1,-1,0,0,1); +}=(-1,0,1,1,0,0,1); ++H=(1,0,-1,-1,0,1,0); +?=(-1,0,1,1,0,1,0); +8=(1,-1,1,0,1,0,0); +O=(1,-1,1,0,0,-1,0); +i=(1,-1,1,0,0,1,0); +’=(1,-1,1,0,0,0,-1); +x=(1,1,0,0,0,1,1); ++O=(1,1,0,0,1,0,-1); ++B=(1,1,0,0,1,1,0);

++M=(1,1,0,0,-1,0,1); ++Q=(1,1,0,0,-1,1,0); +Y=(1,1,0,0,-1,-1,0); +9=(1,1,0,1,0,0,1); +|=(1,1,0,1,0,1,0); K=(1,1,0,1,0,-1,0); +E=(1,1,0,1,1,0,0); +]=(1,-1,1,0,0,0,1); *=(1,1,0,1,-1,0,0); +A=(-1,1,0,1,1,0,0); +o=(1,1,0,-1,0,0,-1); +V=(1,1,0,-1,0,1,0); w=(1,1,0,-1,1,0,0); &=(1,-1,0,-1,1,0,0); +P=(1,1,0,-1,-1,0,0); /=(-1,1,0,1,0,1,0); +_=(1,1,1,0,0,0,1); ’=(1,1,1,0,0,1,0); +I=(1,1,1,0,0,-1,0); \=(1,1,1,0,1,0,0); +$=(-1,1,0,1,0,0,1); +-=(1,-1,0,-1,0,0,1); r=(1,1,1,0,-1,0,0); ++3=(1,1,1,1,0,0,0); ++R=(1,-1,0,1,-1,0,0); +J=(1,-1,0,1,1,0,0); +L=(1,-1,0,1,0,-1,0); s=(1,-1,0,1,0,1,0);]=(1,-1,0,1,0,0,-1); ++I=(1,-1,0,1,0,0,1); +T=(-1,1,0,0,1,1,0); !=(1,1,1,-1,0,0,0); D=(1,-1,0,0,-1,1,0); +‘=(-1,1,0,0,1,0,1); P=(1,-1,0,0,1,-1,0); "=(1,-1,0,0,1,1,0); +r=(1,-1,0,0,0,1,-1); +B=(1,1,-1,0,0,0,-1); ++2=(1,1,-1,0,0,1,0); ++6=(1,1,-1,0,1,0,0); |=(1,1,-1,0,-1,0,0); E=(1,1,-1,1,0,0,0); +s=(1,1,-1,-1,0,0,0).

9-dim

47-16 123PQSUkl, 12BCJLVWb, 13DEMOYab, 1BQRVXdei, 1CPRZafgi, 1DMNWXghj, 1EKLYZcdj, 1IJKTUfhl, 1INOSTcek, 23GHJLVWb, 45EFJKTUb, 46GIJKTUb, 56ABPQSUb, 78ACPQSUb, 79HINOSTb, 89DFQRVXb.
1=(0,0,0,0,0,0,0,0,1); 2=(1,0,0,0,0,0,0,0,0); 3=(0,0,1,0,0,0,0,0,0); 4=(1,-1,1,0,0,0,0,0,1); 5=(-1,1,1,0,0,0,0,0,1); 6=(1,1,1,0,0,0,0,0,-1); 7=(1,-1,1,0,0,0,0,0,-1); 8=(1,1,-1,0,0,0,0,0,-1); 9=(1,1,1,0,0,0,0,0,1); A=(1,0,0,0,0,0,0,0,1); B=(0,1,-1,0,0,0,0,0,0); C=(0,1,1,0,0,0,0,0,0); D=(1,-1,0,0,0,0,0,0,0); E=(1,1,0,0,0,0,0,0,0); F=(0,0,1,0,0,0,0,0,-1); G=(0,1,0,0,0,0,0,0,1); H=(0,1,0,0,0,0,0,0,-1); I=(1,0,-1,0,0,0,0,0,0); J=(0,0,0,1,1,0,0,0,0); K=(0,0,0,1,-1,0,0,0,0); L=(0,0,0,0,0,1,1,0,0); M=(0,0,0,1,0,1,0,0,0); N=(0,0,0,0,1,0,1,0,0); O=(0,0,0,0,1,0,-1,0,0); P=(0,0,0,0,1,-1,0,0,0); Q=(0,0,0,0,1,1,0,0,0); R=(0,0,0,1,0,0,1,0,0); S=(0,0,0,1,0,0,0,0,0); T=(0,0,0,0,0,1,0,0,0); U=(0,0,0,0,0,0,1,0,0); V=(0,0,0,1,-1,1,-1,0,0); W=(0,0,0,1,-1,-1,1,0,0); X=(0,0,0,1,1,-1,-1,0,0); Y=(0,0,0,1,1,-1,1,0,0); Z=(0,0,0,1,1,1,-1,0,0); a=(0,0,0,-1,1,1,1,0,0); b=(0,0,0,0,0,0,0,1,0); c=(1,-1,1,0,0,0,0,-1,0); d=(1,-1,-1,0,0,0,0,1,0); e=(1,1,1,0,0,0,0,1,0); f=(1,-1,1,0,0,0,0,1,0); g=(1,1,-1,0,0,0,0,1,0); h=(1,1,1,0,0,0,0,-1,0); i=(1,0,0,0,0,0,0,-1,0); j=(0,0,1,0,0,0,0,1,0); k=(0,1,0,0,0,0,0,-1,0); l=(0,1,0,0,0,0,0,1,0).

19-5 678125493, 349CDFGHE, EFGHIJ786, ABCD56789, IJAB12789.

7(44)-6 SU(CDEFOQ6), 1G42U(8H95), 1S(AIMSVXbi), 472(acefhK), G72(LWYZdg), 4U(BJ3NPRT).
1=(0,0,0,0,0,0,0,1,0); 2=(0,0,0,0,0,0,1,0,0); 3=(0,0,0,0,1,1,0,0,0); 4=(0,0,0,1,0,0,0,0,1); 5=(0,0,0,1,0,0,0,0,-1); 6=(0,0,0,1,0,0,-1,1,0); 7=(0,0,1,0,0,0,0,1,0); 8=(0,0,1,0,0,1,0,0,0); 9=(0,0,1,0,0,-1,0,0,0); A=(0,0,1,0,-1,0,1,0,0); B=(0,0,1,0,-1,1,1,0,0); C=(0,0,-1,-1,1,1,0,1,1); D=(0,0,1,-1,-1,-1,0,1,1); H=(0,1,0,0,1,0,0,0,0); E=(0,1,0,0,1,-1,1,1,-1); F=(0,1,0,0,1,-1,-1,-1,1); G=(0,1,0,0,-1,0,0,0,0); I=(0,1,0,1,1,1,1,0,1); J=(0,1,0,-1,0,0,0,-1,1); K=(1,-1,1,1,-1,1,0,-1,-1); L=(0,1,0,-1,1,1,0,0,0); M=(0,1,0,-1,1,-1,1,0,-1); N=(0,1,-1,0,0,0,1,1,0); O=(0,1,1,1,0,1,1,0,1); P=(0,1,1,1,1,-1,1,-1,-1); Q=(0,1,1,-1,0,1,-1,0,-1); R=(0,1,-1,0,0,0,1,1,0); S=(0,-1,1,0,1,0,0,0,0); U=(1,0,0,0,0,0,0,0,0); V=(1,0,0,0,0,-1,0,0,1); W=(1,0,0,1,0,1,0,0,1); X=(1,0,0,-1,0,1,0,0,0); Y=(1,0,0,-1,0,-1,0,0,1); Z=(1,0,1,0,0,0,0,-1,-1); a=(1,1,0,0,-1,-1,0,0,0); b=(1,1,1,1,0,0,-1,0,-1); c=(1,1,1,-1,1,1,0,-1,1); d=(-1,1,1,1,1,-1,0,-1,1); e=(1,-1,-1,-1,-1,1,0,1,1); f=(1,1,-1,1,1,1,0,1,-1); g=(1,1,-1,1,1,-1,0,1,-1); h=(1,-1,0,0,1,-1,0,0,0); i=(1,-1,-1,1,0,0,1,0,-1).

MMPHs in Fig. 4(b,c) (**44-6** and **7-6** above) of the main body of the paper illustrate why all vertices are needed for a consistent implementation of MMPHs. Let us assign ‘1’ to S and 2 in Fig. 4(c), i.e. to **7-6**. The hyperedge 4U then provides us with the contextual contradiction since ‘0’ is assigned to both vertices in it. But the hyperedge 4U would not be a hyperedge without all the other vertices (B,J,3,N,P,R,T) necessary for preparing and measuring it, since without them it would be just the orthogonality between 4 and U and would belong to the hyperedge 1G42U, i.e., would not be a separate hyperedge. It is also indicative that Fig. 4(c), i.e. **7-6**, is critical with any or all of a,c,e,f,h,K considered in hyperedge 472 while it ceases to be contextual as soon as we consider any of B,J,3,N,P,R,T in 4U since we can then assign a 3rd ‘1’ to one of them.

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† Passed away on December 9, 2021

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