Pseudoscalar mesons at finite temperature in a separable Dyson-Schwinger model*

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Outline

- Dyson-Schwinger approach to quark-hadron physics
- Separable model at $T = 0$
- Separable model at $T \neq 0$
- Results for pseudoscalar mesons at $T = 0$
- Results for pseudoscalar mesons at $T \neq 0$
- Summary

Gap and BS equations in ladder truncation

$$S_f(p)^{-1} = i\gamma\cdot p + \widetilde{m}_f + \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}\, g^2 D^{\text{eff}}_{\mu\nu}(p-q)\gamma_\mu S_f(q)\gamma_\nu$$

$$\rightarrow S_f(p) = \frac{1}{i\not{p}A_f(p^2)+B_f(p^2)} = \frac{-i\not{p}A_f(p^2)+B_f(p^2)}{p^2A_f(p^2)^2+B_f(p^2)^2} = \frac{-i\not{p}+m_f(p^2)}{p^2+m_f(p^2)^2}$$

$$\lambda(P^2)\Gamma_{f\bar{f}'}(p,P) = -\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D^{\text{eff}}_{\mu\nu}(p-q)\gamma_\mu S_f(q+\tfrac{P}{2})\Gamma_{f\bar{f}'}(q,P)S_f(q-\frac{P}{2})\gamma_\nu$$

- Euclidean space: $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$, $\gamma_\mu^\dagger=\gamma_\mu$, $a\cdot b=\sum_{i=1}^4 a_ib_i$
- P is the total momentum
- meson mass is identified from $\lambda(P^2=-M^2)=1$
- $D^{\text{eff}}_{\mu\nu}(k)$ an "effective gluon propagator" - modeled !

From the gap and BS equations ...

- solutions of the gap equation $\longrightarrow$ the dressed quark mass function

$$m_f(p^2) = \frac{B_f(p^2)}{A_f(p^2)}$$

- propagator solutions $A_f(p^2)$ and $B_f(p^2)$ pertain to confined quarks if

$$m_f^2(p^2) \neq -p^2 \quad \text{for} \quad \text{real} \quad p^2$$

- The BS solutions $\Gamma_{f\bar{f}'}$ enable the calculation of the properties of $q\bar{q}$ bound states, such as the decay constants of pseudoscalar mesons:

$$\begin{aligned} f_{PS}\, P_\mu &= \langle 0|\bar{q}\, \frac{\lambda^{PS}}{2}\, \gamma_\mu\, \gamma_5\, q|\Phi_{PS}(P)\rangle \\ \longrightarrow \quad f_\pi P_\mu &= N_c\, \mathrm{tr_s} \int \frac{d^4q}{(2\pi)^4}\, \gamma_5 \gamma_\mu\, S(q+P/2)\, \Gamma_\pi(q;P)\, S(q-P/2) \end{aligned}$$

Separable model

- To simplify calculations, take the separable form for $D^{\text{eff}}_{\mu\nu}$:

$$D^{\text{eff}}_{\mu\nu}(p-q) \to \delta_{\mu\nu}\, D(p^2, q^2, p\cdot q)$$

$$D(p^2, q^2, p\cdot q) = D_0\ f_0(p^2) f_0(q^2) + D_1\ f_1(p^2)(p\cdot q) f_1(q^2)$$

- two strength parameters D_0, D_1, and corresponding form factors $f_i(p^2)$. In the separable model, gap equation yields

$$\begin{aligned} B_f(p^2) &= \widetilde{m}_f + \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \\ \left[A_f(p^2) - 1\right] p^2 &= \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{(p\cdot q) A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \,. \end{aligned}$$

- This gives $B_f(p^2) = \widetilde{m}_f + b_f\ f_0(p^2)$ and $A_f(p^2) = 1 + a_f\ f_1(p^2)$, reducing to nonlinear equations for constants b_f and a_f.

A simple choice for 'interaction form factors' of the separable model:

- $f_0(p^2) = \exp(-p^2/\Lambda_0^2)$
- $f_1(p^2) = [1 + \exp(-p_0^2/\Lambda_1^2)]/[1 + \exp((p^2 - p_0^2))/\Lambda_1^2]$
  gives good description of pseudoscalar properties if the interaction is strong enough for realistic DChSB, when $m_{u,d}(p^2 \sim small) \sim$ the typical constituent quark mass scale $\sim m_\rho/2 \sim m_N/3$.
- Another simplification: for the separable interaction, the solution for the pseudoscalar BS amplitude reduces to just two terms:

$$\Gamma_{PS}(q;P) = \gamma_5 \left[iE_{PS}(P^2) + \not{P} F_{PS}(P^2) \right] \, f_0(q^2)$$

Extension to $T \neq 0$

- At $T \neq 0$, the quark 4-momentum $p \longrightarrow p_n = (\omega_n, \vec{p})$, where $\omega_n = (2n+1)\pi T$ are the discrete ($n = 0, \pm 1, \pm 2, \pm 3, \ldots$) Matsubara frequencies, so that $p_n^2 = \omega_n^2 + \vec{p}^{\,2}$.
- Gap equation solution for the dressed quark propagator

$$S_f(p_n, T) = [i\vec{\gamma} \cdot \vec{p}\, A_f(p_n^2, T) + i\gamma_4 \omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)]^{-1}$$

$$= \frac{-i\vec{\gamma} \cdot \vec{p}\, A_f(p_n^2, T) - i\gamma_4 \omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)}{\vec{p}^{\,2}\, A_f^2(p_n^2, T) + \omega_n^2\, C_f^2(p_n^2, T) + B_f^2(p_n^2, T)} \,.$$

- There are now three amplitudes due to the loss of $O(4)$ symmetry, and at sufficiently high $T \geq T_d$ denominator CAN vanish. $\longrightarrow$ For $T \geq T_d$ quarks can be deconfined!

Extension to $T \neq 0$

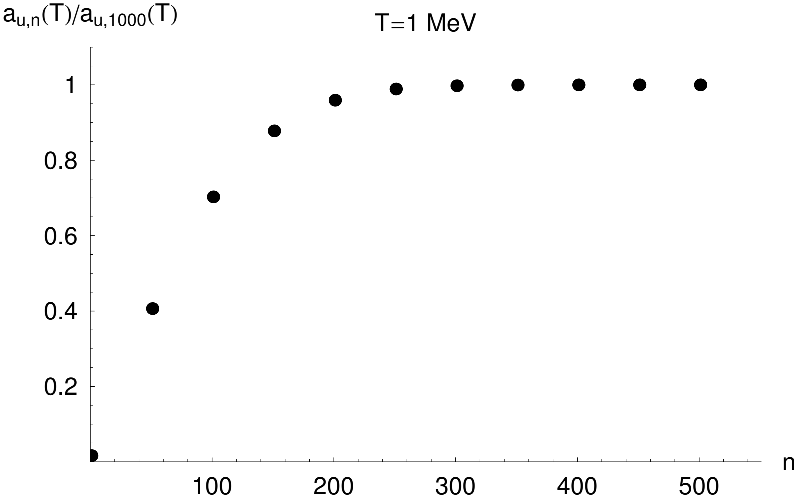
- The solutions have the form $B_f = \widetilde{m}_f + b_f(T) f_0(p_n^2)$, $A_f = 1 + a_f(T) f_1(p_n^2)$, and $C_f = 1 + c_f(T) f_1(p_n^2)$

$$
\begin{aligned}
a_f(T) &= \frac{8D_1}{9} T \sum_n \int \frac{d^3p}{(2\pi)^3} f_1(p_n^2)\, \vec{p}^{\,2} \left[1 + a_f(T) f_1(p_n^2)\right] d_f^{-1}(p_n^2, T) \\
c_f(T) &= \frac{8D_1}{3} T \sum_n \int \frac{d^3p}{(2\pi)^3} f_1(p_n^2)\, \omega_n^2 \left[1 + c_f(T) f_1(p_n^2)\right] d_f^{-1}(p_n^2, T) \\
b_f(T) &= \frac{16D_0}{3} T \sum_n \int \frac{d^3p}{(2\pi)^3} f_0(p_n^2) \left[\widetilde{m}_f + b_f(T) f_0(p_n^2)\right] d_f^{-1}(p_n^2, T)
\end{aligned}
$$

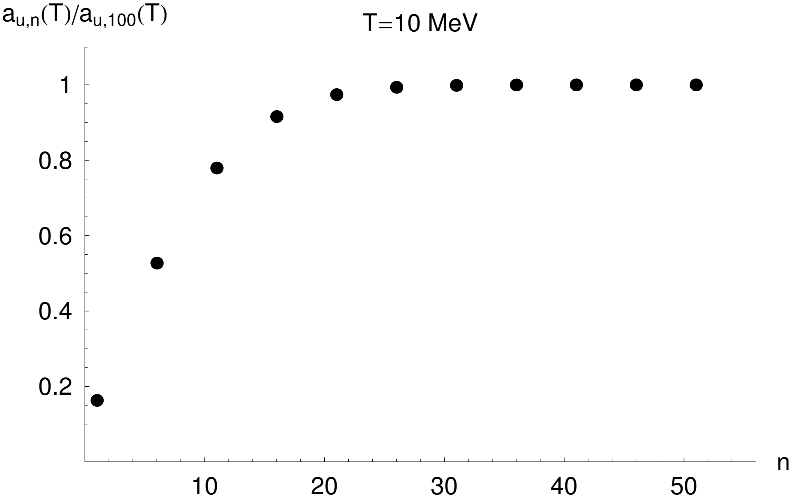
- where $d_f(p_n^2, T)$ is given by

$$
d_f(p_n^2, T) = \vec{p}^{\,2} A_f^2(p_n^2, T) + \omega_n^2 C_f^2(p_n^2, T) + B_f^2(p_n^2, T)
$$

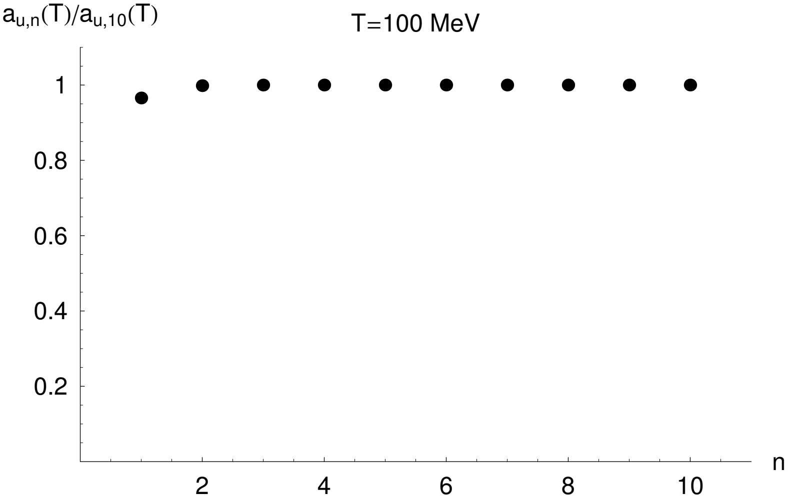
Matsubara sums



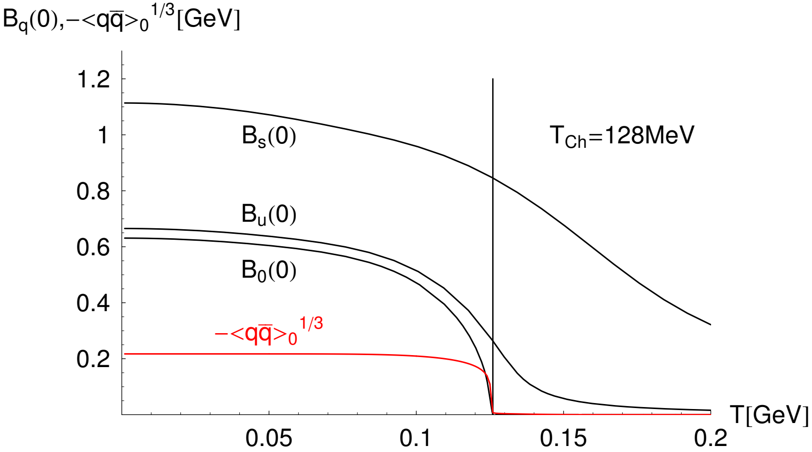
Matsubara sums



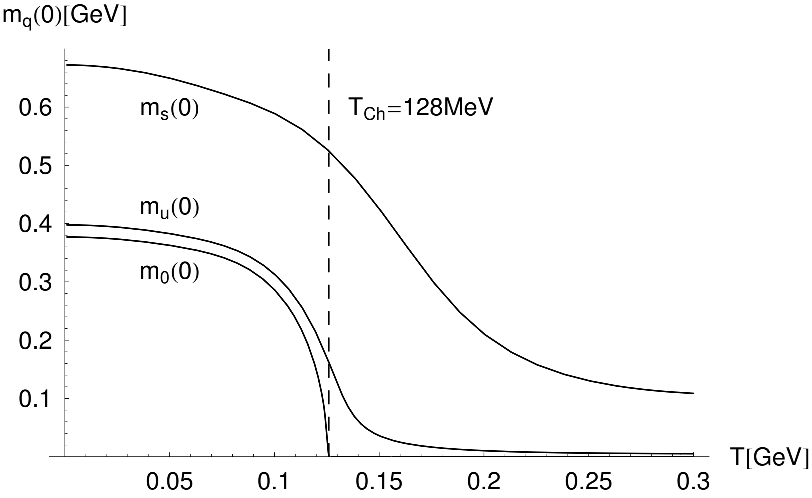
Matsubara sums



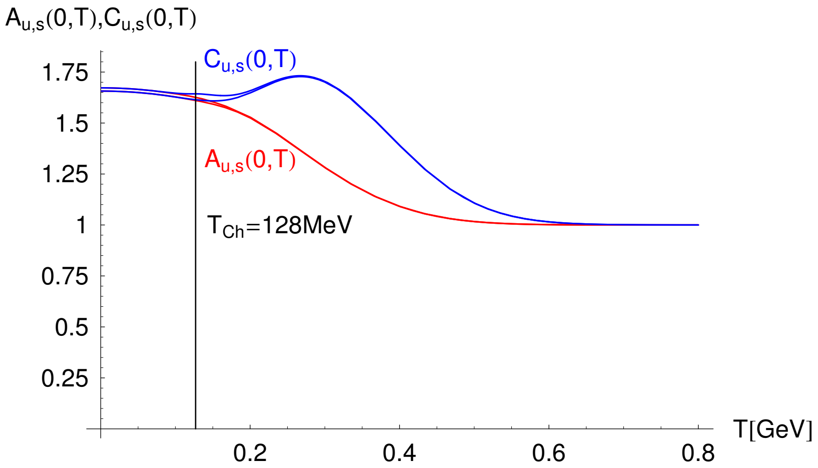
Chiral symmetry restoration at $T = T_{Ch}$



Chiral symmetry restoration at $T = T_{Ch}$



Violation of $O(4)$ symmetry with T



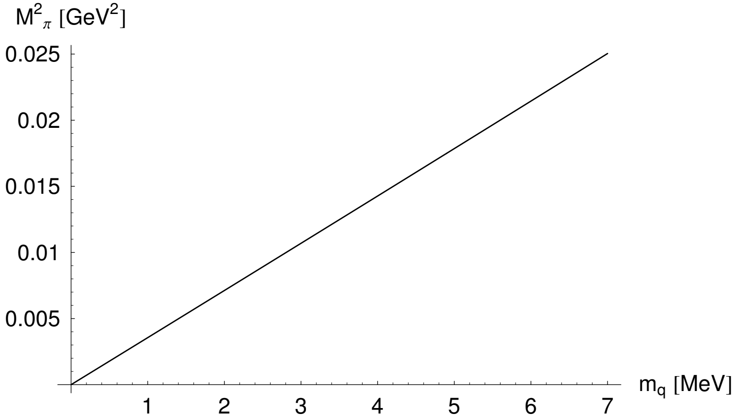
Model results at $T = 0$

- Model parameter values reproducing experimental data:
- $\widetilde{m}_{u,d} = 5.5$ MeV, $\Lambda_0 = 758$ MeV, $\Lambda_1 = 961$ MeV, $p_0 = 600$ MeV, $D_0\Lambda_0^2 = 219$, $D_1\Lambda_1^4 = 40$ (fixed by fitting M_π, f_π, M_ρ, $g_{\rho\pi^+\pi^-}$, $g_{\rho e^+e^-}$ $\longrightarrow$ predictions $a_{u,d} = 0.672$, $b_{u,d} = 660$ MeV, i.e., $m_{u,d}(p^2)$, $\langle \bar{u}u \rangle$)
- $\widetilde{m}_s = 115$ MeV (fixed by fitting M_K $\longrightarrow$ predictions $a_s = 0.657$, $b_s = 998$ MeV, i.e., $m_s(p^2)$, $\langle \bar{s}s \rangle$, $M_{s\bar{s}}$, f_K, $f_{s\bar{s}}$)
- *Summary of results (all in GeV) for $q = u, d, s$ and pseudoscalar mesons without the influence of gluon anomaly:*

PS	M_{PS}	f_{PS}	$-\langle \bar{q}q \rangle_0^{1/3}$	$m_q(0)$
π	0.140	0.092	0.217	0.398
K	0.495	0.110		
$s\bar{s}$	0.685	0.119		0.672

Model results at $T = 0$

- GMOR



$\eta - \eta'$ complex

$$
\begin{aligned}
M_\eta^2 &= \frac{1}{2}\left[\ M^2_{\eta_{NS}} +\ M^2_{\eta_S} - \sqrt{(M^2_{\eta_{NS}} -\ M^2_{\eta_S})^2 + 8\beta^2 X^2}\right] \\
&= \frac{1}{2}\left[M_\pi^2 + M_{s\bar{s}}^2 + \beta(2+X^2) - \sqrt{(M_\pi^2+2\beta-M_{s\bar{s}}^2-\beta X^2)^2 + 8\beta^2 X^2}\right] \\
M_{\eta'}^2 &= \frac{1}{2}\left[\ M^2_{\eta_{NS}} +\ M^2_{\eta_S} + \sqrt{(M^2_{\eta_{NS}} -\ M^2_{\eta_S})^2 + 8\beta^2 X^2}\right] \\
&= \frac{1}{2}\left[M_\pi^2 + M_{s\bar{s}}^2 + \beta(2+X^2) + \sqrt{(M_\pi^2+2\beta-M_{s\bar{s}}^2-\beta X^2)^2 + 8\beta^2 X^2}\right]
\end{aligned}
$$

$$X = f_\pi / f_{s\bar{s}}$$

$$\beta\,(2 + X^2) = m_\eta^2 + m_{\eta'}^2 - 2m_K^2 = \frac{2N_f}{f_\pi^2}\chi$$

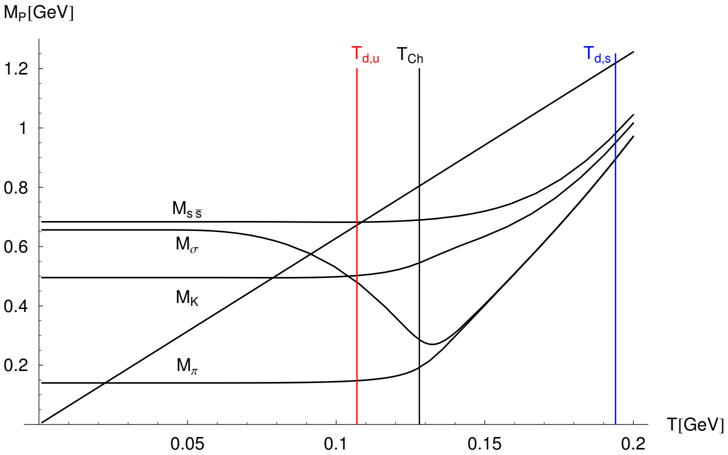
Results on $\eta - \eta'$ complex at $T = 0$

	$\beta_{\rm fit}$	$\beta_{\rm latt.}$	Exp.
θ	-12.22°	-13.92°	
M_η	548.9	543.1	547.75
$M_{\eta'}$	958.5	932.5	957.78
X	0.772	0.772	
3β	0.845	0.781	

- masses are in units of MeV, 3β in units of GeV^2 and the mixing angles are dimensionless.
- $\beta_{\rm latt.}$ was obtained from $\chi(T = 0) = (175.7\ \mathrm{MeV})^4$ using Witten-Veneziano relation.

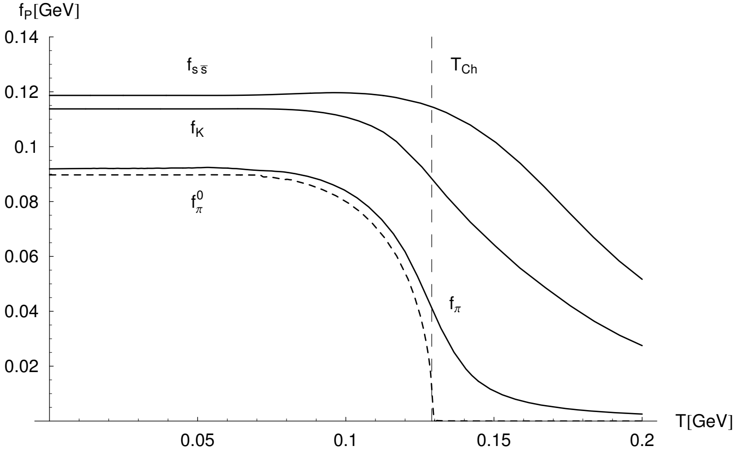
Model results at $T \neq 0$

- T-dependence of the masses of light mesons: $\pi, K, s\bar{s}, \sigma$



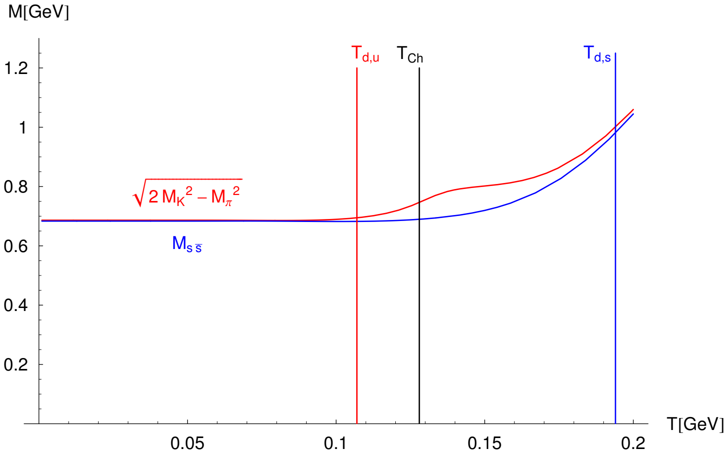
Model results at $T \neq 0$

- T-dependence of pseudoscalar decay constants f_P

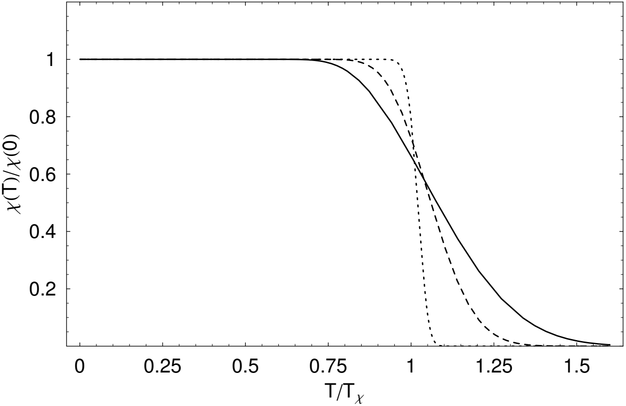


Model results at $T \neq 0$

- $m^2_{s\bar{s}} \approx 2m^2_K - m^2_\pi$ due to GMOR

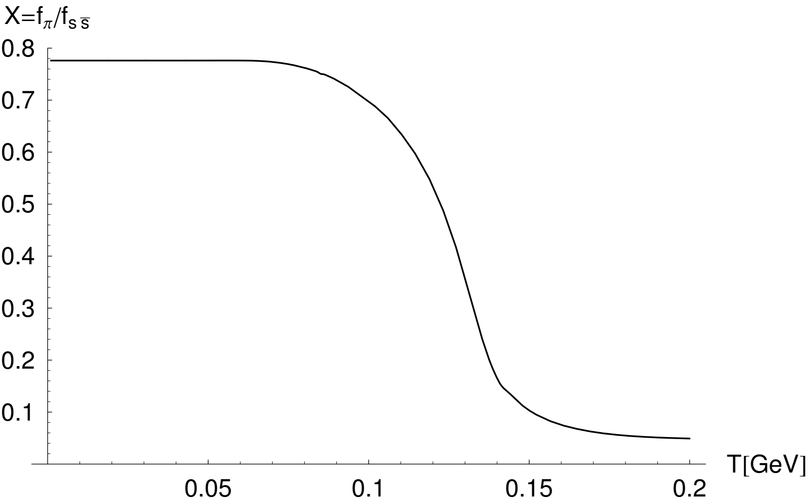


Topological susceptibility

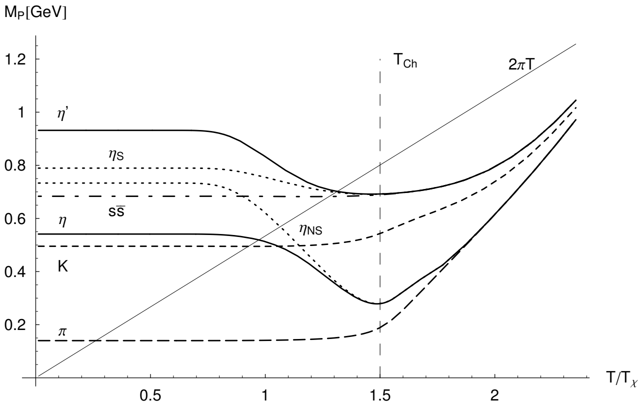


YM - solid curve, SU(3) quenched - dashed, $N_f = 4$ QCD - dotted

$X = f_\pi / f_{s\bar{s}}$

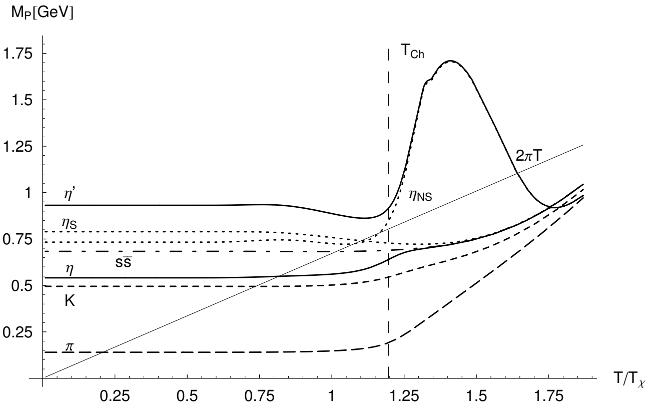


Results for pseudoscalar nonet at $T \neq 0$



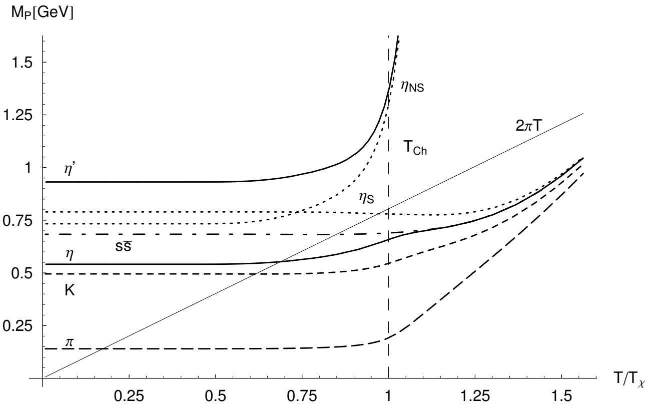
$T_\chi = 2/3 T_{\rm Ch}$, YM susceptibility

Results for pseudoscalar nonet at $T \neq 0$



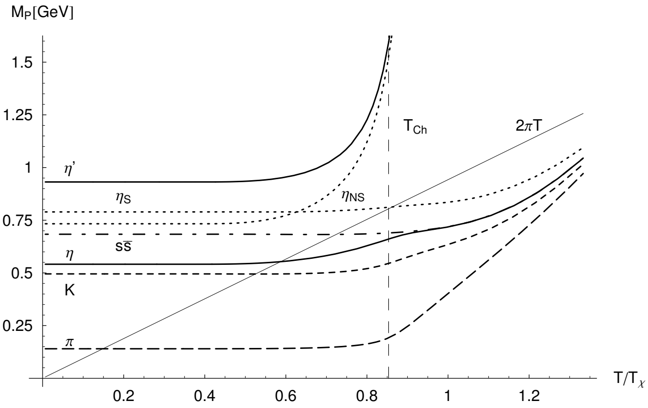
$T_\chi = 0.836 T_{\mathrm{Ch}}$, YM susceptibility

Results for pseudoscalar nonet at $T \neq 0$



$T_\chi = T_{\rm Ch}$, YM susceptibility

Results for pseudoscalar nonet at $T \neq 0$



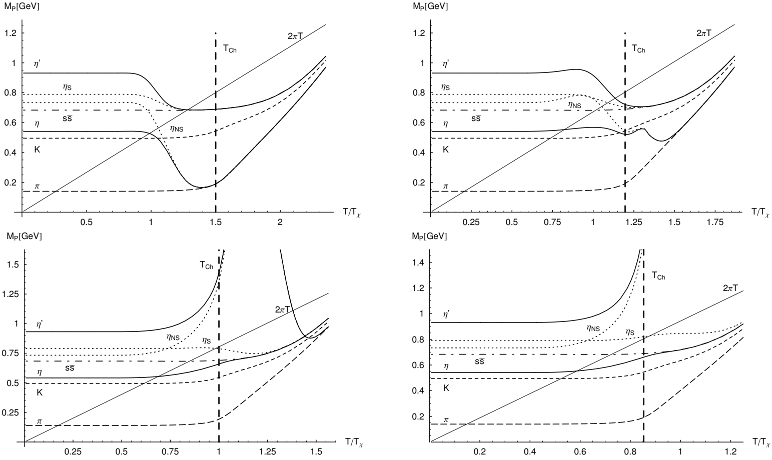
$T_\chi = 1.17 T_{\rm Ch} = 150\,{\rm MeV}$, YM susceptibility

Results for pseudoscalar nonet at $T \neq 0$

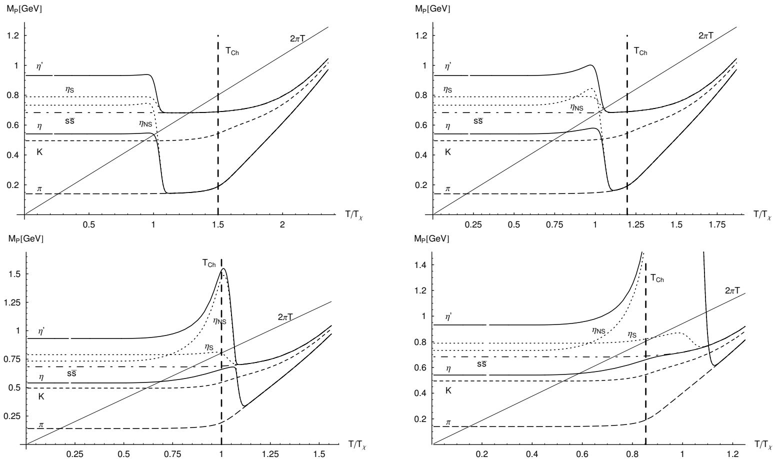
Animation for range of T_χ

Movie

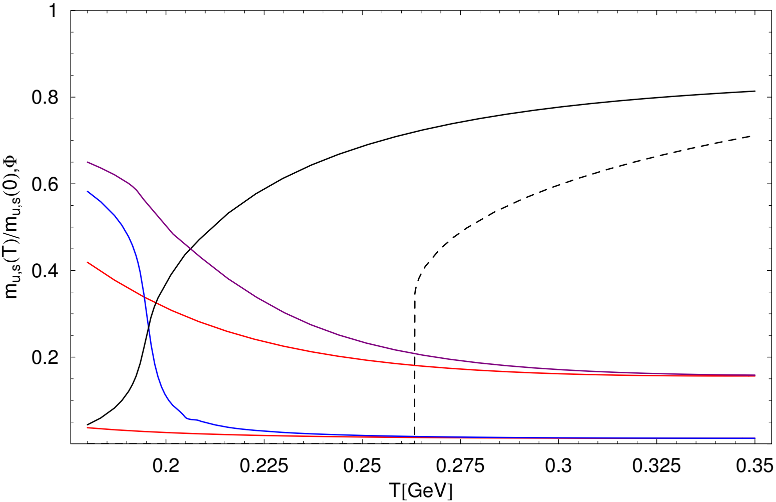
PS nonet at $T \neq 0$, SU(3) quenched susceptibility



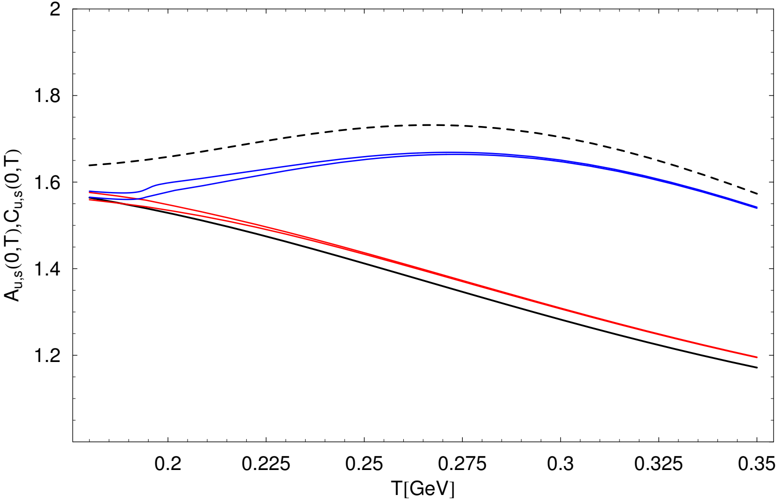
PS nonet at $T \neq 0$, $N_f = 4$ QCD susceptibility



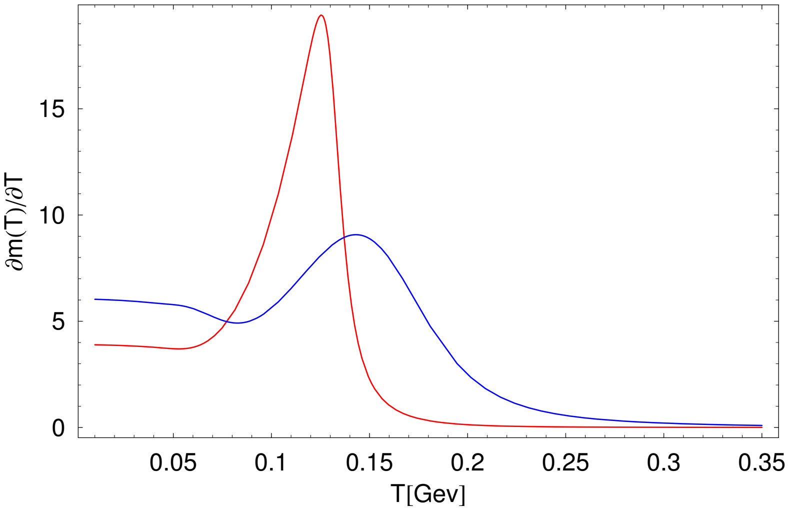
Polyakov loop, preliminary



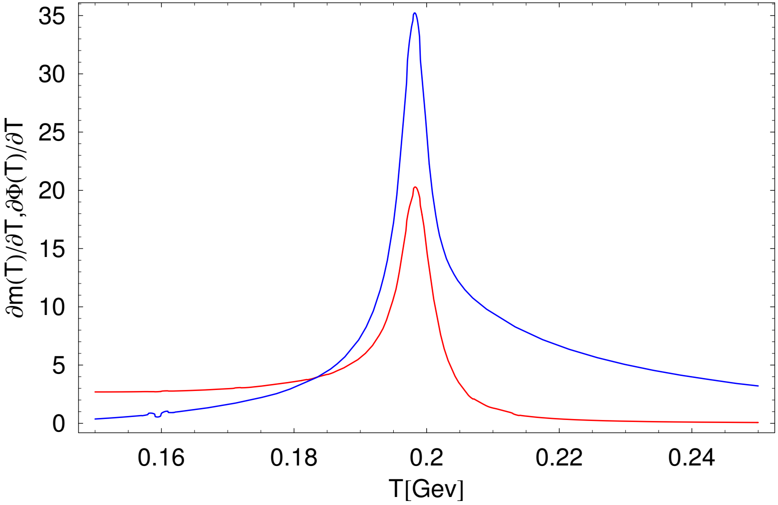
Polyakov loop, preliminary



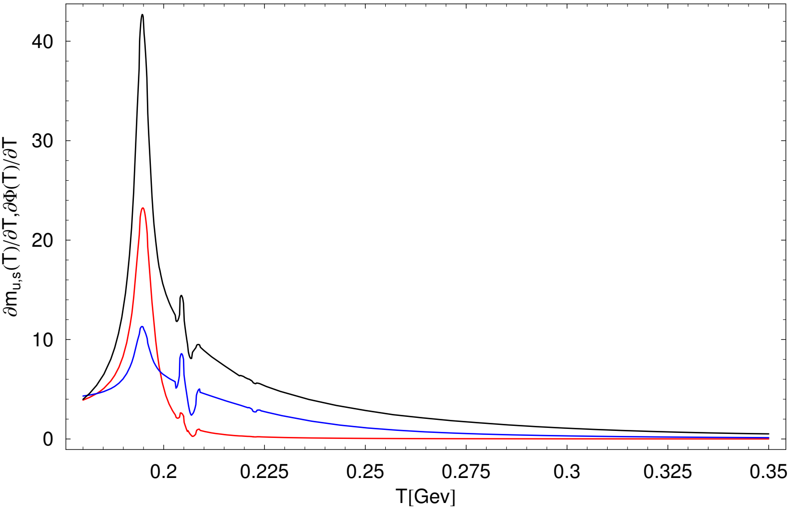
Order parameters without PL



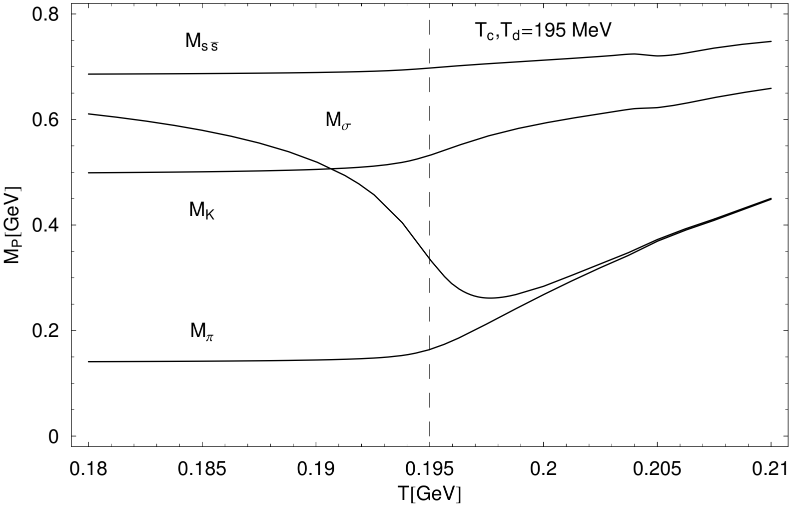
Polyakov loop, preliminary



Polyakov loop, preliminary



Polyakov loop, preliminary



Summary

- Sketched Dyson-Schwinger approach to quark-hadron physics & a convenient concrete model
- Results for dressed quarks and pseudoscalar mesons at $T = 0$
- Results for dressed quarks and pseudoscalar mesons at $T \neq 0$