Production of photons to the order α in out of equilibrium QGP

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Heavy ion collisions at ultrarelativistic energies - possibility to examine the fundamental properties of QCD:

- asimptotic freedom

- deconfinement

- quarks and gluons - quasifree particles in experimental conditions (?)

$\Rightarrow$ Model of quark-gluon plasma (QGP) is based on asimptotic freedom of QCD

QCD at high temperature and matter density:

- thermal field theory
- out of equilibrium field theory

Necessary for calculation of QGP properties - knowledge of the initial state
$\Rightarrow$ Parton cascade models:

- parton collisions lead to forming of hot plasma of quarks and qluons

- termalisation after $0.1 - 1$ fm/c (?) and hydrodynamic expansion

Emission of hadron particles follows after rehadronisation and freeze-out

Possible phase coexistence: QGP and hadronic phase (hadron gas, DCC)

Photons and leptons: emited in all phases of the collision, leave the plasma without further interactions

$\Rightarrow$ clean signals of QGP forming in experiments

Photon production

Photon production in ultrarelativistic heavy ion collision - standard approach - different contributions:

- photons emited in hard parton interactions similar to QCD Compton scattering, anihilation, bremsstrahlung

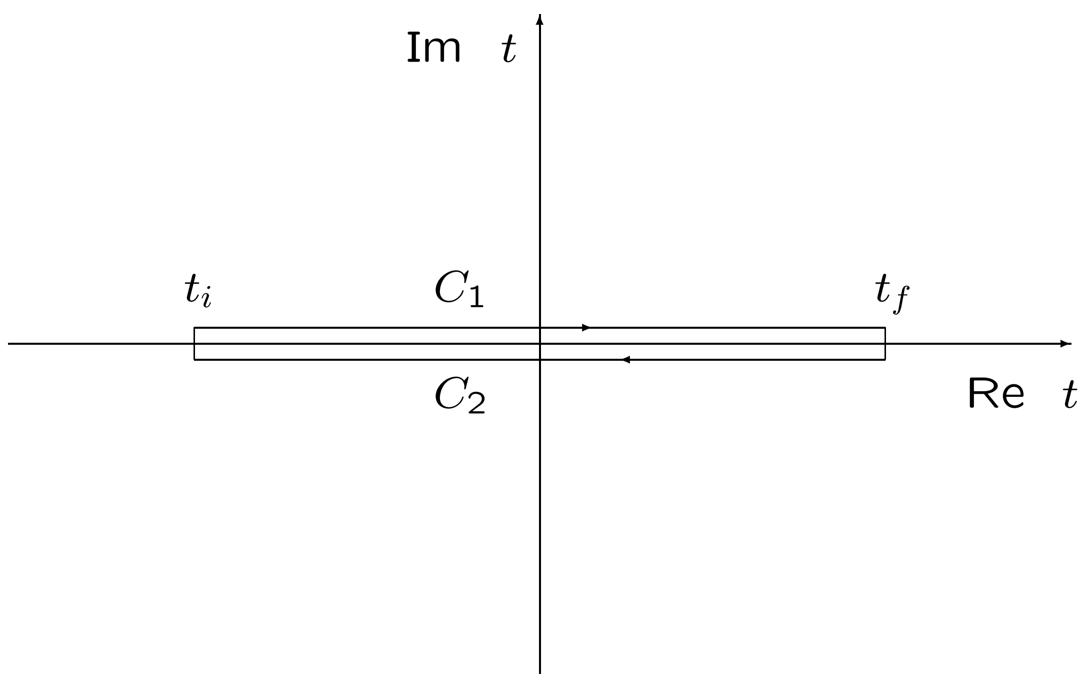
- photons emited in quark-gluon collisions in QGP

- photons emited by hadronic particles (π^0, ρ, ω, ...) after rehadronisation

Model of photon production in out of equilibrium field theory

- system is prepared at t_i in initial state
- at $t_i = 0$ time evolution starts and ends at t_f

Time evolution defined along the conture $C = C_1 \cup C_2$ in complex plane



Time ordering T_c is defined on the conture C and equal to zero elsewhere

Averages of operators (density operator ρ)

$$\langle O(t) \rangle = \mathrm{Tr} \rho O(t)$$

Two-point Green function

$$G^c(x, x') = -i \langle \ \mathsf{T}_c \phi(x) \phi(x') \ \rangle$$

$G^c(x, x')$ for t, t' on $C = C_1 \cup C_2$ connected with components of the Keldysh space $G_{ij}(x, x') \quad (i, j = 1, 2)$

Limit $t_i \to -\infty$, $t_f \to \infty \Rightarrow$ nontrivial connection with the Keldysh approach

Projected functions

Two-point function $G(x,y)$ with four-vector variables x, y, time components inside the interval $t_i < x_0,\ y_0 < \infty$

Wigner variables

$$X = \frac{x+y}{2}, \quad s = x - y$$

$$G(x,y) = G(X + \frac{s}{2}, X - \frac{s}{2})$$

Lower boundary for $x_0,\ y_0 \Rightarrow 0 < X_0$ and $-2X_0 < s_0 < 2X_0$

The values of $G(X + \frac{s}{2}, X - \frac{s}{2})$ for (X, s) not satisfying $0 < X_0$ and $-2X_0 < s_0 < 2X_0$ are physically irrelevant

Definition of time ordering T_c sets them to zero outside intervals $0 < X_0$, $-2X_0 < s_0 < 2X_0$

$G(X + \frac{s}{2}, X - \frac{s}{2})$ is written using the projection operator

$$G(X + \frac{s}{2}, X - \frac{s}{2})$$

$$= \theta(X_0)\theta(2X_0 - s_0)\theta(2X_0 + s_0)$$

$$\times \bar{G}(X + \frac{s}{2}, X - \frac{s}{2})$$

Outside the carrier of the projection operator values of $\bar{G}(X + \frac{s}{2}, X - \frac{s}{2})$ are arbitrary

Wigner transform of projected function - Fourier transform w.r.t. $(s_0, \vec{s})$

$$G(p_0, \vec{p}, X)$$

$$= \int d^4 s\ e^{i(p_0 s_0 - \vec{p}\cdot\vec{s})} G(X + \frac{s}{2}, X - \frac{s}{2})$$

$$G(X + \frac{s}{2}, X - \frac{s}{2})$$

$$= \frac{1}{(2\pi)^4} \int d^4 p\ e^{-i(p_0 s_0 - \vec{p}\cdot\vec{s})} G(p_0, \vec{p}, X)$$

Homogeneity in space coordinates excludes dependence on $\vec{X}$

Projection operator has a simple transform

$$P_{X_0}(p_0, p_0')$$

$$= \frac{1}{2\pi}\theta(X_0)\int_{-2X_0}^{2X_0} ds_0 \; e^{is_0(p_0-p_0')}$$

$$= \frac{1}{\pi}\theta(X_0)\frac{\sin\left(2X_0(p_0-p_0')\right)}{p_0-p_0'}$$

Important property for energy conservation in the limit $X_0 \to \infty$

$$\lim_{X_0\to\infty} P_{X_0}(p_0, p_0') = \delta(p_0 - p_0')$$

For finite X_0 (finite time) $\Rightarrow$ energy nonconservation

Function $\bar{G}(X + \frac{s}{2}, X - \frac{s}{2}) = \bar{G}(s_0, \vec{s})$ follows from the projected function in the limit $X_0 \to \infty$

$$\lim_{X_0 \to \infty} G(X + \frac{s}{2}, X - \frac{s}{2}) = \bar{G}(s_0, \vec{s})$$

Important property of projected functions $\Rightarrow$ transform of the projection operator induces X_0 dependence

$$G_{X_0}(p_0, \vec{p}) = \left[P_{X_0} G_\infty \right] (p_0, \vec{p})$$

$$= \int_{-\infty}^{\infty} dp'_0 P_{X_0}(p_0, p'_0) G_\infty(p'_0, \vec{p})$$

Important examples of projected functions are retarded, avanced and Keldysh component of free propagators

For further analysis analytic properties in the $X_0 \to \infty$ limit of Wigner transforms of projected functions (WTPF) are important

We define the following properties corresponding to R (A) components:
(1) function of p_0 is analytic above (below) real axis
(2) function goes to zero as $|p_0|$ approaches infinity in the upper (lower) semiplane

Convolution products of projected functions

$$C = A_1 * A_2 * \ldots * A_{n-1} * A_n$$

For convolution products of n projected functions it is important that at least $n-1$ functions satisfy assumptions (1) and (2)

Order is also important: the retarded functions should be on the right, the avanced on the left, and the functions neither avanced nor retarded in the middle

If these conditions are fulfiled:

$$C_{X_0}(p_0, \vec{p})$$

$$= \int dp_{0,1} P_{X_0}(p_0, p_{0,1}) \prod_{j=1}^{n} A_{j,\infty}(p_{0,1}, \vec{p})$$

$\Rightarrow$ Convolution products of Wigner transforms of projected functions are Wigner transforms of projected functions (WTPF)

But propagators and self-energies in the Schwinger-Dyson equations appear in different order

$\Rightarrow$ terms that are not WTPF appear in Schwinger-Dyson equations

Schwinger-Dyson equations

Schwinger-Dyson equations for R, A and K components of the propagator

$$\mathcal{G}_R = G_R + iG_R * \Sigma_R * \mathcal{G}_R$$

$$\mathcal{G}_A = G_A + iG_A * \Sigma_A * \mathcal{G}_A$$

$$\mathcal{G}_K = G_K + iG_R * \Sigma_K * \mathcal{G}_A$$

$$+iG_K * \Sigma_A * \mathcal{G}_A + iG_R * \Sigma_R * \mathcal{G}_K$$

Formal solutions for retarded and advanced component

$$\mathcal{G}_R = G_R * (1 - i\Sigma_R * G_R)^{-1}$$

$$\mathcal{G}_A = G_A * (1 - i\Sigma_A * G_A)^{-1}$$

R and A components of the resummed propagator are Wigner tranforms of projected functions (WTPF)

Keldysh component of the resummed propagator

$$\mathcal{G}_K = \mathcal{G}_R * \Big(h(p_0, \omega_p)(G_A^{-1} - G_R^{-1})$$

$$+ i\Sigma_K) * \mathcal{G}_A$$

Keldysh component of self-energy does not satisfy assumptions (1) and (2)

One-loop aproximation to Σ_K can be decomposed into parts satisfying (1) and (2) as retarded and advanced functions

$$\Sigma_K = -\Sigma_{K,R} + \Sigma_{K,A}$$

But Schwinger-Dyson equation for K component of the propagator contains retarded components on the left from the advanced components

$\Longrightarrow$ stepping out of the space of Wigner transforms of projected functions (WTPF)

Equal time two-point functions and opservables

To study single-particle opservables: reduction of two-point functions to equal time ($x_0 = y_0 = t \Rightarrow X_0 = t$, $s_0 = 0$)
$\Rightarrow$ it is obtained by inverse Wigner transform

$$G(t, 0, \vec{p}) = \frac{1}{2\pi} \int dp_0 G_{X_0=t}(p_0, \vec{p})$$

Average number of particles with impulse $\vec{p}$ is connected to equal time K component of the propagator

$$\langle 2N_{\vec{p}}(t) + 1 \rangle = \frac{\omega_p}{2\pi} \int dp_0 G_{K,t}(p_0, \vec{p})$$

Other single-particle opservables are generated with the help of $\langle N_{\vec{p}}(t)\rangle$

For projected functions and bare fields

$$\langle 2N_{\vec{p}}(t)+1\rangle = \frac{\omega_p}{2\pi}\int dp_0' G_{K,\infty}(p_0',\vec{p})$$

$$= 1 + 2f(\omega_p)$$

This is completely determined by its form in the $X_0 \to +\infty$ limit

Equal time K component of the propagator in the single self-energy insertion aproximation $G_K = G_K^0 + G_K^1 + \ldots$

$$\langle 2N_{\vec{p}}(t) + 1 \rangle$$

$$= \langle 2N_{\vec{p}}^0(t) + 1 \rangle + \langle 2N_{\vec{p}}^1(t) \rangle + \ldots$$

$$= 1 + 2f(\omega_p) + \frac{\omega}{2\pi} \int dp_0 G^1_{K,X_0}(p_0, \vec{p})$$

Time dependence of single-particle opservables is described by equal time two-point functions

All terms coming from WTPF are constants in time
$\Rightarrow$ non-WTPF terms generate the time dependence

Number of photons in QGP

Average photon number with impulse $\vec{p}$ produced in QGP

$$\langle N_{\vec{p}}(t)\rangle = \frac{d\mathcal{N}(t)}{d^3p d^3x}(2\pi)^3$$

$$= \frac{\omega_p}{4\pi}\int dp_0[\mathcal{D}_{t,K}(p_0,\vec{p}) - \mathcal{D}_{0,K}(p_0,\vec{p})]$$

Assumption: initial state (at $t_i = 0$) contains no photons

$\Rightarrow$ "prompt" photons leave the medium

Phase space photon number density

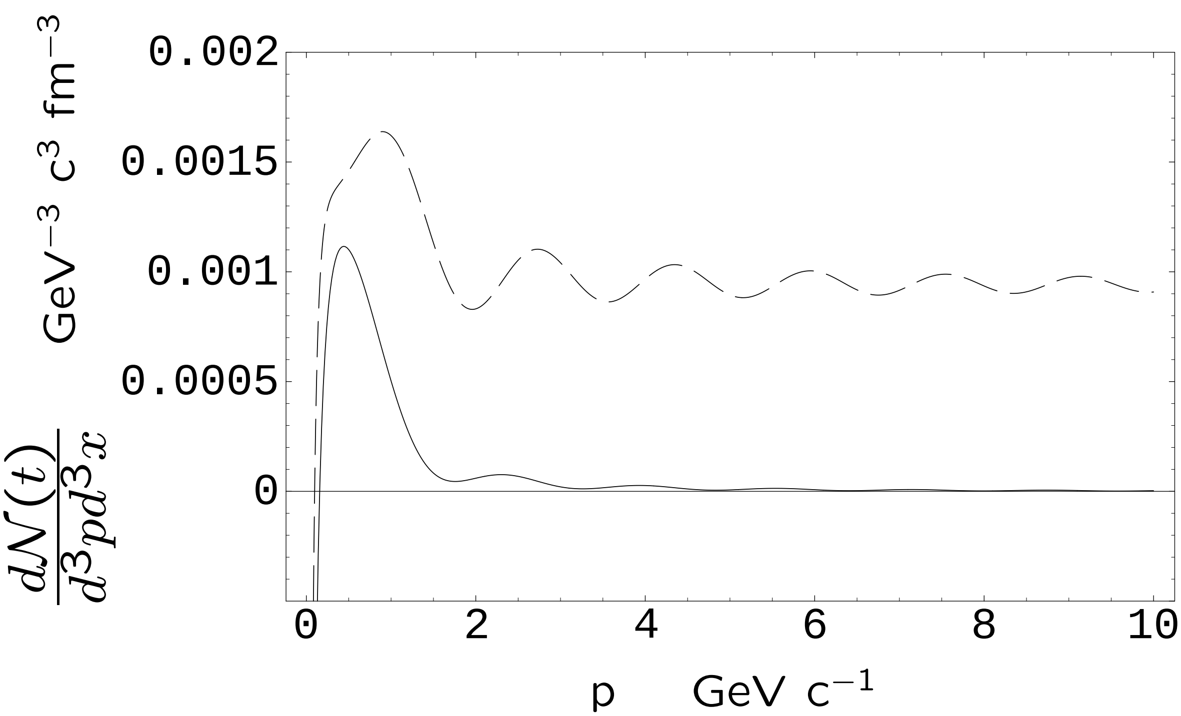
$$\frac{d\mathcal{N}(t)}{d^3p\,d^3x} = \frac{\langle N_{\vec{p}}(t)\rangle}{(2\pi)^3}$$

$$= -\frac{2}{\pi(2\pi)^3}\frac{p}{2}\left(\int_{-\infty}^{\infty} dp_0 \mathcal{P}\frac{\mathrm{Im}\tilde{\Sigma}_{\infty,K,R}(p_0,\vec{p})}{(p_0^2-\vec{p}^2)^2}\right.$$

$$\left[1-\cos(p_0-p)t+\frac{p-p_0}{p}\sin tp_0 \sin tp\right]$$

$$\left.+2\pi\frac{1}{4p^3}\sin^2 tp \sum_{\lambda=\pm 1}\lambda \mathrm{Re}\tilde{\Sigma}_{\infty,K,R}(\lambda p,\vec{p})\right)$$

Photon number density with vacuum contribution (dashed line) and photon number density without vacuum contribution (full line) vs. photon impulse p at $t = 10$ fm/c. Parameter T is equal 0.2 GeV. Quark masses (u i d) are set equal to zero.



- photon number density is negative at small impulse $p \ll T$ (region where resummation is necessary - HTL)

- at large impuse total photon number and energy are infinite

This is a consequence of the choice of initial conditions: initial states are eigenstates on the Fock space of non-interacting hamiltonian
$\Rightarrow$ regularization is necessary

Regularization

Problem of initial conditions: in one-loop aproximation (order α in coupling constant) total energy emited through photon field is infinite

Without the formal solution, finite results at the order α can be achieved, by considering four basic types of QCD plasma

1. vacuum plasma with intial quark and antiquark distribution functions equal to zero ($f_+ = 0$ and $f_- = 0$)

2. quark plasma with $f_+ \neq 0$ and $f_- = 0$

3. antiquark plasma with $f_- \neq 0$ and $f_+ = 0$

4. quark-antiquark plasma with $f_+ \neq 0$and $f_- \neq 0$

For "bare" initial conditions all four types of plasma emit infinite amount of energy at the order α in the coupling constant

Had we prepared "dressed" initial conditions only quark-antiquark plasma should emit photons at the order α in the coupling constant

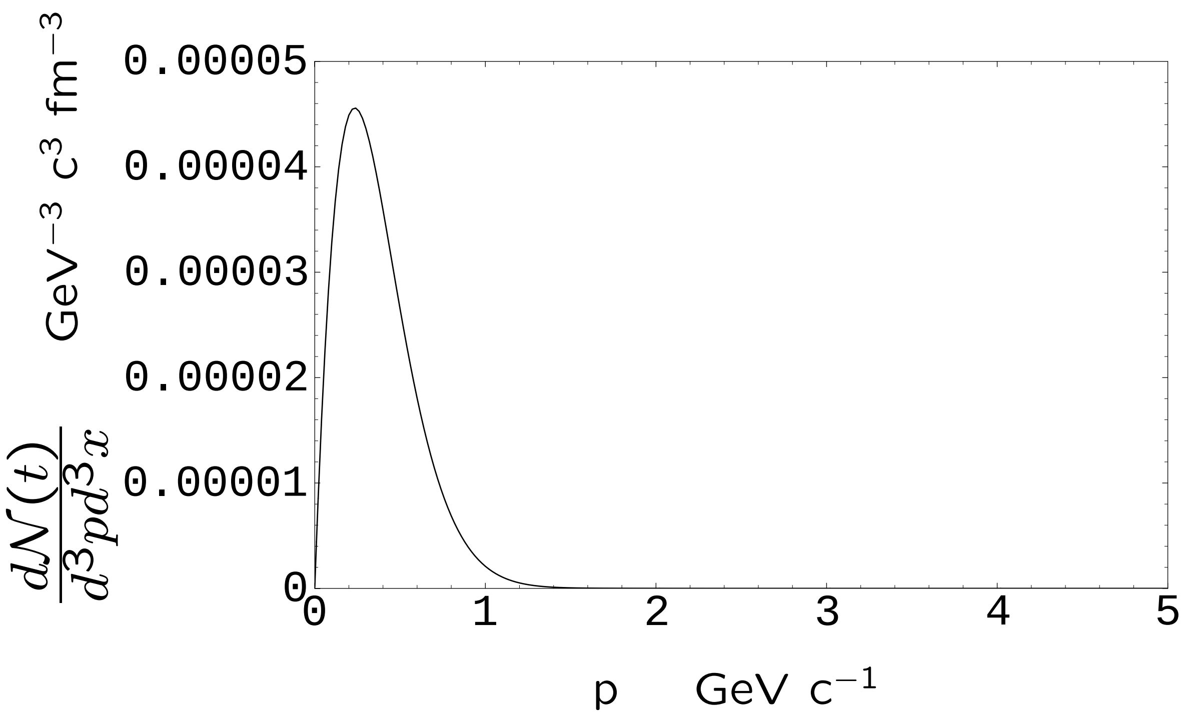
Quark-antiquark plasma contains other three types of plasma and reduces on them as special cases

By substracting these contributions to average photon number regularized expression is obtained which gives finite total energy

$$N_{f_+,f_-,reg}(\vec{p},t) = N_{f_+,f_-}(\vec{p},t)$$

$$-N_{f_+,0}(\vec{p},t) - N_{0,f_-}(\vec{p},t) + N_{0,0}(\vec{p},t)$$

Regularized phase space photon number density for parameter $T = 0.2$ GeV and $t = 10$ fm/c vs. photon impulse

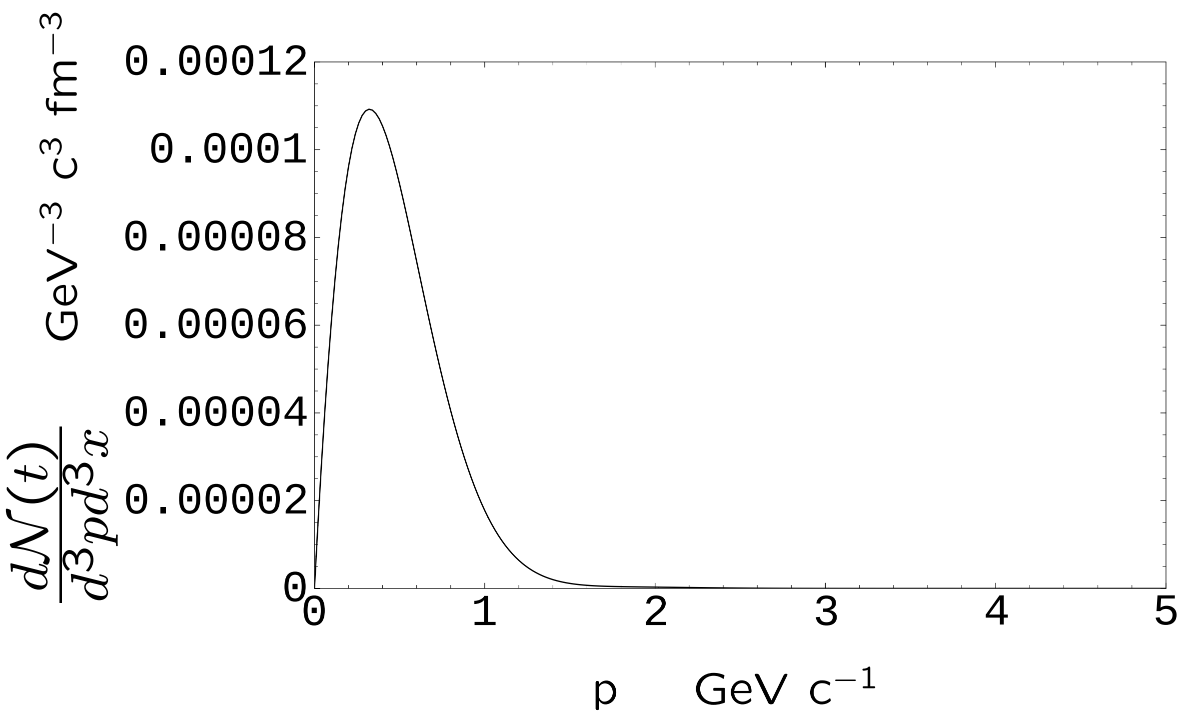


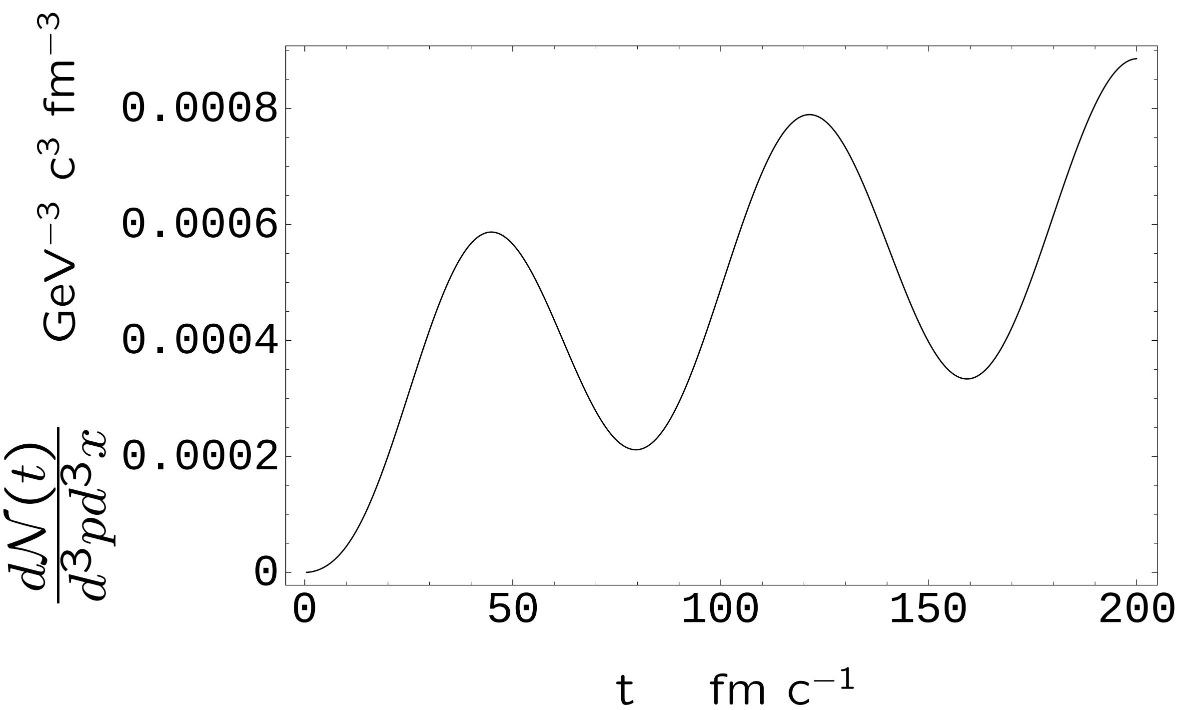
- function is positive at small impulse $(p \ll T)$

- function falls exponentially at large impulse

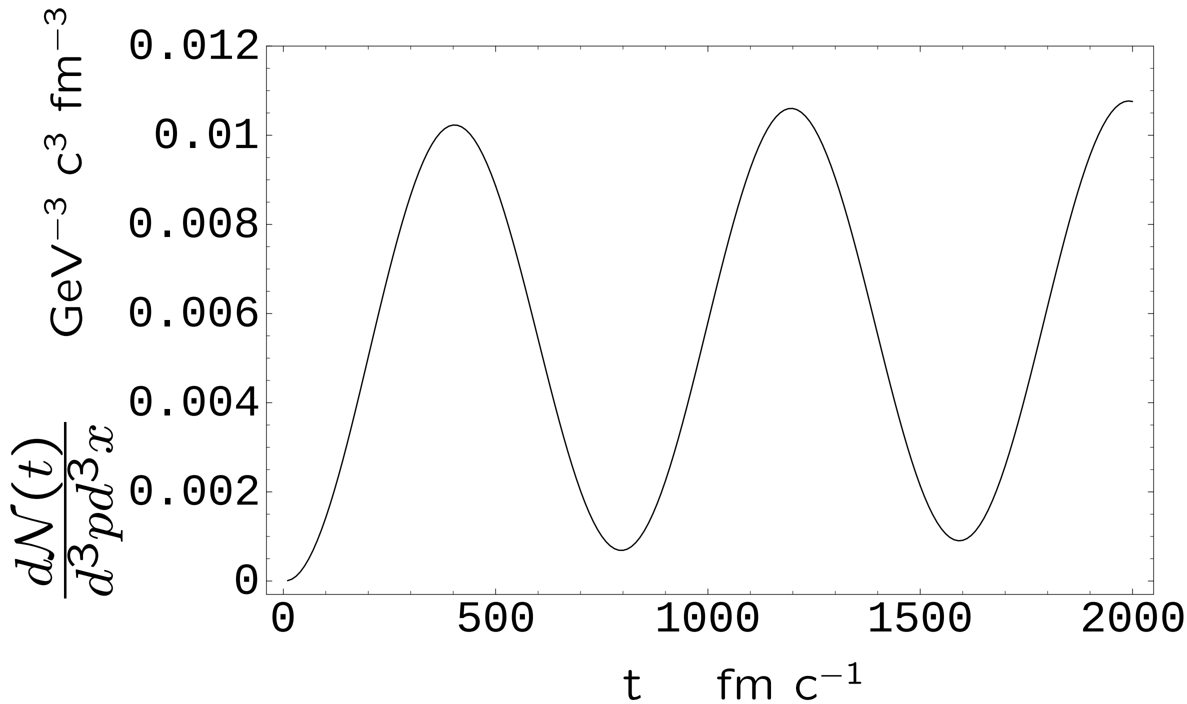
$\Rightarrow$ regularization gives finite total emited photon energy

Regularized phase space photon number density for parameter $T = 0.3$ GeV and $t = 10$ fm/c vs. photon impulse





Photon number density at $p = 0.2$ GeV/c vs. time. Parameter T is equal 0.2 GeV.



Photon number density at $p = 0.02$ GeV/c vs. time. Parameter T is equal 0.2 GeV.

I. Dadić, *Out of Equilibrium Thermal Field Theories: Finite time after Switching on the Interaction and Wigner Transforms of Projected Functions*, Phys. Rev. D63, 025011 (2001); hep-ph/9910337

I. Dadić, *Pinching Phenomenon: Central Feature in out of Equilibrium Thermal Field Theories*, hep-ph/0103025

I. Dadić, *Out-of-Equilibrium TFT - Energy Nonconservation at Vertices*, Nucl.Phys. A702, 356-360 (2002)