The O(N) linear sigma model at NLO of the large N approximation using the Dyson-Schwinger formalism

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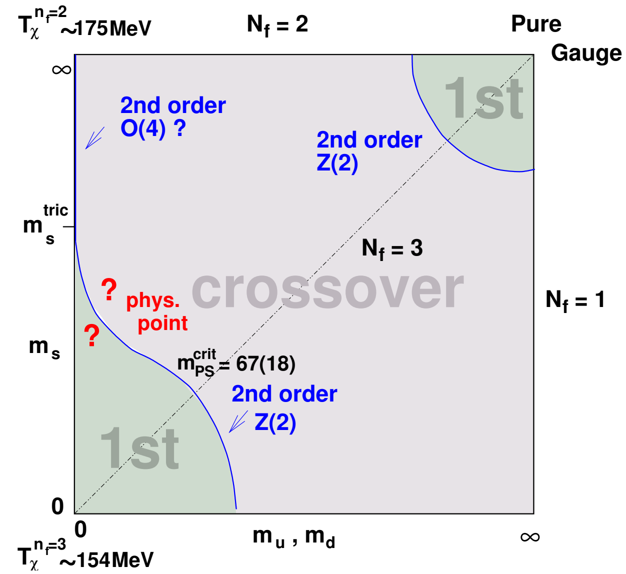
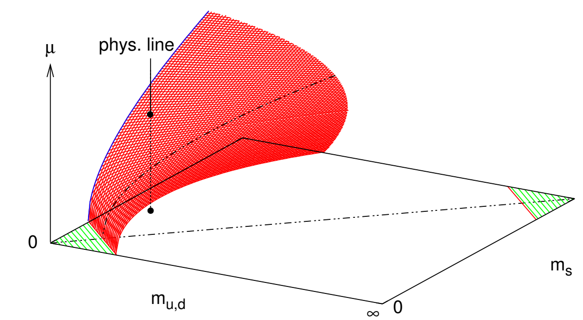
- Motivation for resummation in effective models
  – properties of particle excitations in matter
  – phase transition of strongly interacting matter as function of $T, \mu, m_{u,d}, m_s$
- Dyson-Schwinger formalism and Ward identities
- $O(N)$ model at leading and next-to-leading order of the large N approximation
- Iteration of the vertex function and renormalization of the $O(N)$ model at NLO
- Conclusions

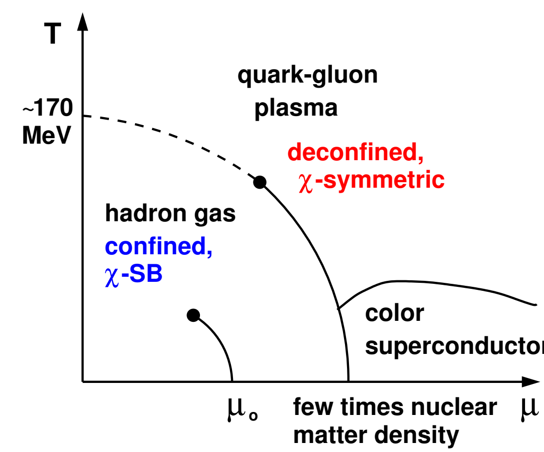
Collaborators: T. Herpay, A. Jakovác, A. Patkós, P. Szépfalusy

Particle physics: phase transition of strongly interacting matter

- order of the phase transition as a function of quark masses chemical potentials
- shape of the transition line, location (?) of CEP
- interplay between chiral and $U_A(1)$ symmetry restoration
- change of meson properties (mass, width) across the transition
- soft mode(s) at CEP
  - scalar density fluctuation and/or
  - sigma mode
- nonequilibrium dynamics near TCP/CEP







Relevance for heavy ion collisions

Consistently resummed quantum field theoretical equations are required to understand

- the thermalization of the QGP

- properties of the quark-gluon plasma in terms of transport coefficients (conductivity, viscosity)

- propagation of heavy quarks in the plasma

- changes in the meson properties above T_c

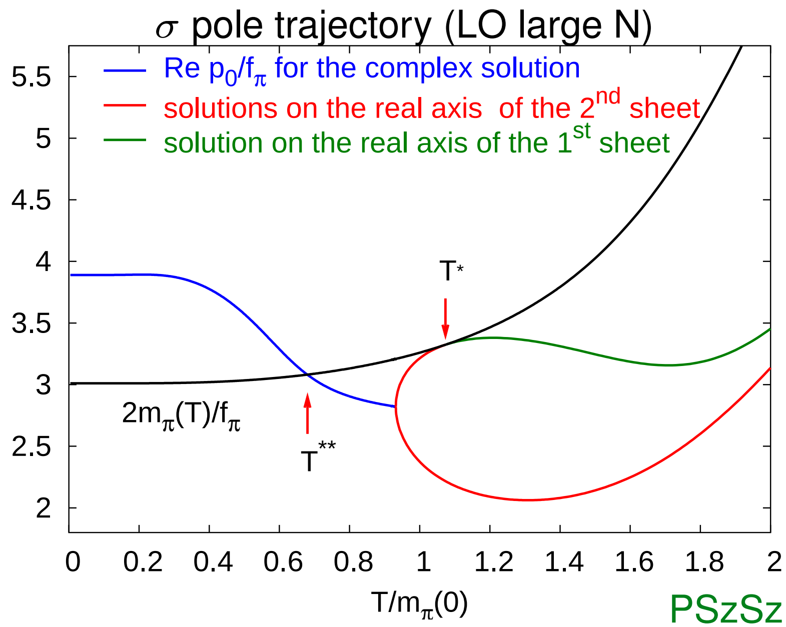
Properties of the σ pole at finite temperature

change in the ground state is reflected upon the properties of σ

$\longrightarrow$ indicative of the degree of chiral symmetry breaking

m_σ decreases during chiral symmetry restoration $\longrightarrow$ phase space of $\sigma \longrightarrow 2\pi$ decay squeezes

$\longrightarrow$ chance to see σ as a sharp resonance Hatsuda & Kunihiro PRL55:158



PSzSz PRD66:116004

$T^{**} \approx 0.69 m_\pi(0) = 96.6$ MeV

real part of the pole goes below the threshold

$T^{*} \approx 1.07 m_\pi(0)$: sigma becomes stable

decay width vanishes at $T_{\rm real} \in (T^{**}, T^{*})$

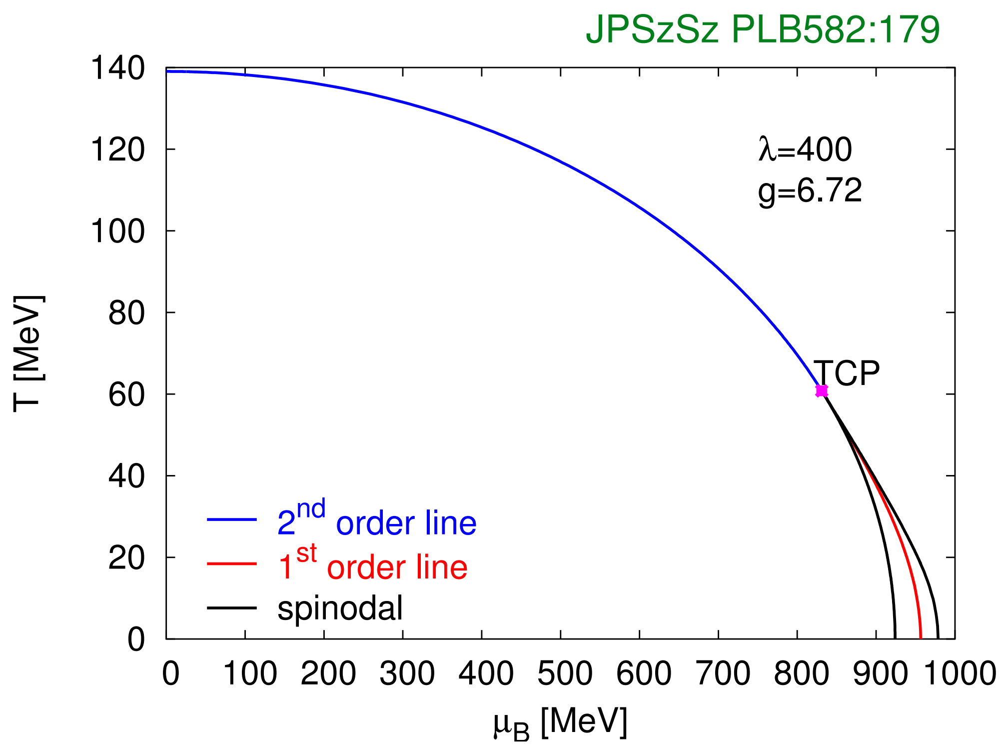
suppression of the $\sigma \longrightarrow 2\pi$ decay channel

- QCD in the composite operator formalism: $T^{**} = 0.95 T_c \simeq 98$ MeV

Barducci *et al.*, PRD59:114024 ($T \neq 0$); PRD59:114024 ($\rho \neq 0$)

Does the scenario changes at NLO ?

$\mu - T$ phase diagram



Chiral Constituent Quark Model
in the LO of the large N
$N_{\mathrm{f}} = 2 \; m_{u,d} = 0 \;\; m_s = \infty$

- qualitatively correct
- the (pseudo)critical transition line can be described by a parabola
- location of TCP analytically determined

2^{nd} order+spinodal line

$$m^2 + \left(\frac{\lambda}{6} + g^2 N_c\right)\frac{T_c^2}{12} + \frac{g^2 \mu_q^2}{2\pi^2 N_f} N_c = 0$$

2^{nd} order line ends when

$$\frac{\lambda}{6} + \frac{g^4 N_c}{4\pi^2}\left[\frac{\partial}{\partial n}\Big(\mathrm{Li}_n\left(\frac{1}{z}\right) + \mathrm{Li}_n\left(z\right)\Big)\Big|_{n=0} - \ln\frac{cT_c}{M_0}\right] = 0 \qquad z = -e^{\frac{\mu_q}{T_c}}$$

$\Rightarrow T_{TCP}$, μ_{TCP}

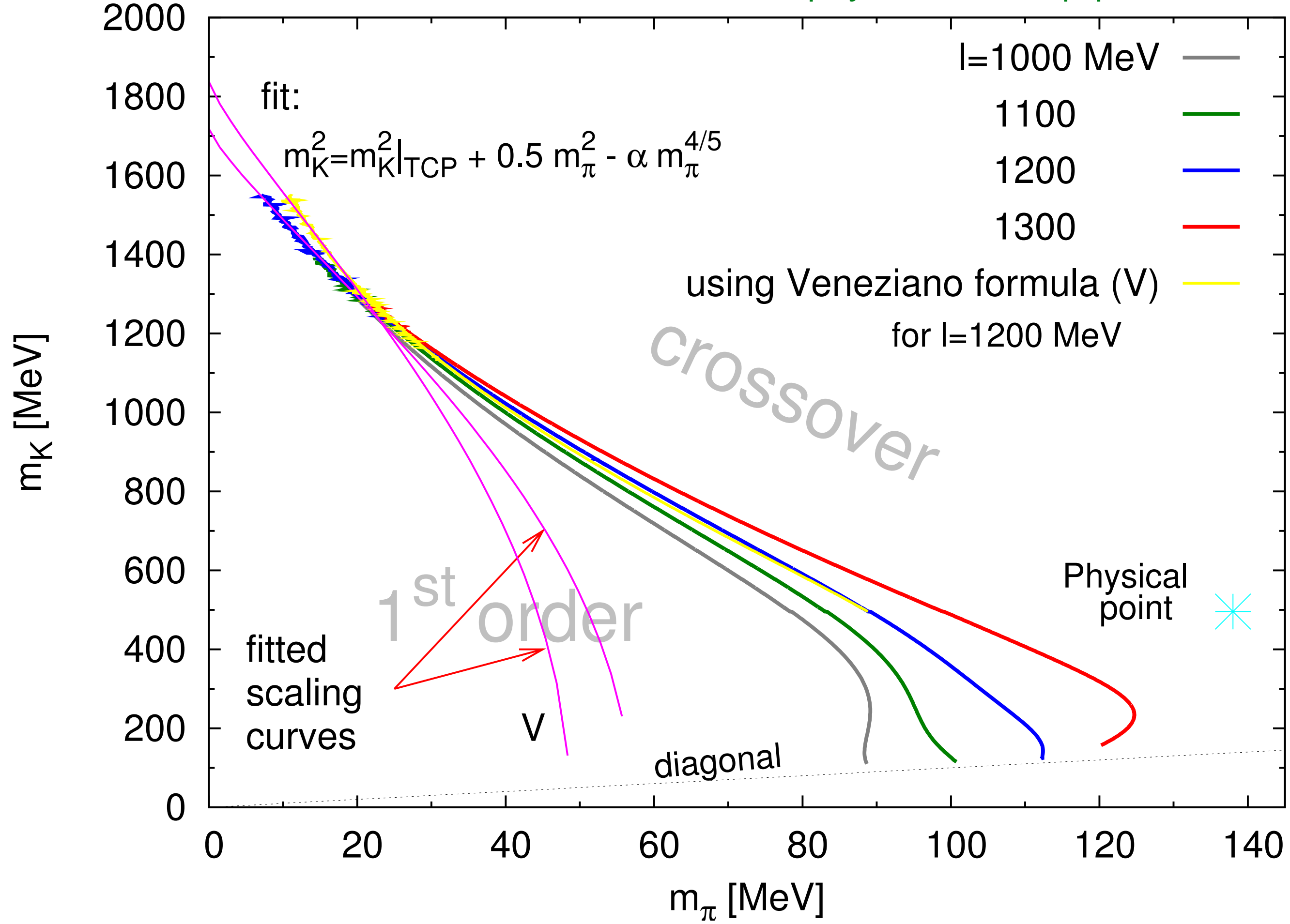
fermions treated perturbatively at one-loop order but an infinite subset of diagrams contribute at $\mathcal{O}(1/\sqrt{N}) \Longrightarrow$ resummation is needed

Phase boundary with one-loop parametrization of the LσM

estimate for $m_\pi = m_K$: $m_\pi^c \in (90, 130)$ MeV

location of TCP: $m_K^{TCP} \in (1700, 1850)$MeV $\Rightarrow m_s = (13 - 15) \times m_s^{\text{phys}}$

T. Herpay, Zs. Sz., hep-ph/0604086



message: the scaling region of TCP sets in far away from the physical point close to the $m_{u,d} = 0$ axis to test the result a self-consistent approximation (Dyson-Schwinger, 2PI) would be useful

ambition: study of the phase boundary along the diagonal of the $m_{u,d} - m_s$-plane using $SU(N) \times SU(N)$ linear sigma model in the large N approximation

challenge since 1981

footnote in A. J. Paterson, Nucl. Phys. **B 190**, (1981), 188-204:

"This result indicates that between 2 and 4 dimensions, the onset of symmetry breaking crosses over from first to second order in the $SU(n) \times SU(n)$ σ models [a]".

[a] It would be interesting to further study both the linear and non-linear $SU(n) \times SU(n)$ σ models in the $n \to \infty$ limit' Unfortunately, this limit does not appear to yield a tractable calculation for either model at present.

Derivation of Dyson-Schwinger equations

technically the functional integral of a functional derivative vanishes

$$\int \mathcal{D}\Phi \frac{\delta e^{i[S+\Phi\cdot J]}}{\delta\Phi(x)} = 0 \qquad \longrightarrow \int \mathcal{D}\Phi \left[\frac{\delta S(\Phi)}{\delta\Phi(x)} + J(x)\right] e^{i[S(\Phi)+\Phi\cdot J]} = 0,$$

physically infinite set of integro-differential equations for Green-functions

Generating equation:

using $\left(\frac{\delta}{i\delta J(x)}\right)^n Z[J] = \int \mathcal{D}\Phi\ \Phi^n(x) e^{i[S+\Phi\cdot J]}$ with $Z[J] = \int \mathcal{D}\Phi e^{i[S+\Phi\cdot J]}$

Z: $\left[\frac{\delta S}{\delta\Phi(x)}\left(\frac{\delta}{i\delta J(x)}\right) + J(x)\right] Z[J] = 0 \qquad \frac{\delta S(\Phi)}{\delta\Phi(x)} = -\left[(\partial^2 + m^2)\Phi(x) + \frac{\lambda}{6}\Phi^3(x)\right]$

$W[J] = -i \ln Z[J]$: $\left[\frac{\delta S}{\delta\Phi(x)}\left(\frac{\delta}{i\delta J(x)} + \frac{\delta W[J]}{\delta J(x)}\right) + J(x)\right] \mathbf{1} = 0$

$\Gamma[\Phi_A] = W[J_A] - \Phi_A J_A$: $\frac{\delta\Gamma[\Phi]}{\delta\Phi_A} = \frac{\delta S}{\delta\Phi_A}\left(\Phi_A + G_{AB}\frac{\delta}{\delta\Phi_B}\right)\mathbf{1}$

$$\Phi_A = \frac{\delta W[J]}{\delta J_A}, \quad \frac{\delta\Gamma[\Phi]}{\delta\Phi_A} = -J_A, \quad \frac{\delta}{\delta J_A} = \frac{\delta\Phi_B}{\delta J_A}\frac{\delta}{\delta\Phi_B} = iG_{AB}\frac{\delta}{\delta\Phi_B}, \quad \frac{\delta^2\Gamma[\Phi]}{\delta\Phi_A\delta\Phi_B} = iG^{-1}_{AB},$$

Derivation of Ward identities

technically generator-functional invariant under infinitesimal symmetry transf.

the classical action is invariant under symmetry transformations

$$\Phi_i(x) \longrightarrow \Phi_i(x) + i\omega_\alpha t^\alpha_{ij}\Phi_j(x)$$

the measure of the functional integral invariant under orthogonal transformations

$$0 = \delta Z[J] = \int \mathcal{D}\Phi \int d^4x J_i(x) t^\alpha_{ij}\Phi_j(x) e^{i[S+\int d^4x \Phi_k(x) J_k(x)]}$$

physically manifestation of classical symmetries at quantum level

$\Longrightarrow$ provides relations between different n-point functions

$$\int d^4x\ t^\alpha_{ij} J_i(x)\frac{\delta Z[J]}{\delta J_j(x)} = 0 \qquad \Longrightarrow \qquad \int d^4x\ t^\alpha_{ij}\Phi_i(x)\frac{\delta \Gamma[\Phi]}{\delta \Phi_j(x)} = 0$$

O(N) model: $$i\sum_{m,n}\int d^4x \frac{\delta\Gamma[\Phi]}{\delta\Phi_n(x)}(t^{ab})_{nm}\Phi_m(x) = 0$$

$(t^{ab})_{nm} = \delta_{am}\delta_{bn} - \delta_{bm}\delta_{an}$ generators of the rotation in a plain

acting with $\dfrac{\delta^2}{\delta\Phi_i(y)\delta\Phi_0(z)}$ and taking the result at the minimum $\to \Phi_m(x) = v\sqrt{N}\delta_{m0}$

$$v\sqrt{N}\Gamma_{\pi\pi\sigma}(0,p,-p) = iG^{-1}_\sigma(p) - iG^{-1}_\pi(p)$$

Large-N approach to the $O(N)$ model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^a \partial^\mu \Phi^a - \frac{1}{2} m^2 (\Phi^a)^2 + \frac{\lambda}{24N}(\Phi^a)^2(\Phi^b)^2 + \sqrt{N} h \Phi^0$$

large-N expansion makes strongly self-coupled theory tractable

coupling rescaled such as

– energy density be proportional to the number of d.o.f. per site $\sim N$
– mass stays finite $\sim N^0$.

external field h determines the pion mass

in the broken symmetry phase: $\Phi_a \to \sqrt{N} v \delta_{a0} + \Phi_a \;\; \sigma = \Phi_0, \pi_i = \Phi_i \;\;\; i = 1, \ldots N-1$

$$\begin{aligned} L[\sigma, \pi_i] &= U_0(v) - \sigma E_0(v) \\ &+ \frac{1}{2}\left[(\partial\sigma)^2 + (\partial\vec{\pi})^2\right] - \frac{1}{2} m_{\sigma 0}^2 \sigma^2 - \frac{1}{2} m_{\pi 0}^2 \vec{\pi}^2 - \frac{\lambda v}{6\sqrt{N}} \sigma \rho^2 - \frac{\lambda}{24N}\rho^4 + \sqrt{N}\sigma h \end{aligned}$$

$$\rho^2 = \sigma^2 + \vec{\pi}^2 \qquad U_0(v) = N\left[\tfrac{\lambda}{24} v^4 + \tfrac{1}{2} m^2 v^2\right], \qquad E_0(v) = \sqrt{N}\left[\tfrac{\lambda}{6} v^3 + m^2 v\right]$$

The $O(N)$ model at leading order

Master equations for DS formalism

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi_A} = \frac{\delta S}{\delta\Phi_A}\left(\Phi_A + G_{AB}\frac{\delta}{\delta\Phi_B}\right) \qquad A = 0 \text{ or } \quad i$$

$$\frac{\delta S}{\delta\pi_i} = -\left[(\partial^2 + m^2_{\pi,0})\pi_i + \frac{\lambda}{3\sqrt{N}}v\sigma\pi_i + \frac{\lambda}{6N}\pi_i\pi_j^2 + \frac{\lambda}{6N}\pi_i\sigma^2\right]$$

$$\frac{\delta S}{\delta\sigma} = -\left[(\partial^2 + m^2_{\sigma,0})\sigma + \frac{\lambda}{2\sqrt{N}}v\sigma^2 + \frac{\lambda}{6\sqrt{N}}v\pi_i^2 + \frac{\lambda}{6N}\sigma^3 + \frac{\lambda}{6N}\sigma\pi_i^2 + E_0(v) - \sqrt{N}h\right]$$

Equation of state: $\frac{\delta\Gamma[\Phi]}{\delta\sigma} = -J_0 = 0$ $\qquad$ $v\sqrt{N}\left[m^2 + \frac{\lambda}{6}v^2 + \frac{\lambda}{6}\int_p G_\pi(p) - \frac{h}{v}\right] = 0$

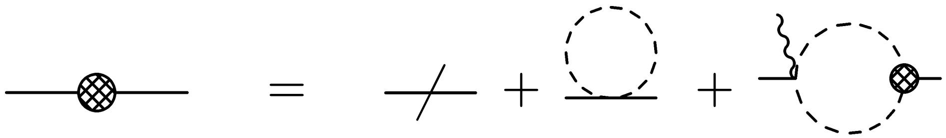
Pion propagator: $\frac{\delta^2\Gamma[\Phi]}{\delta\pi_i\delta\pi_j} = iG^{-1}_{ij}$ $\qquad$ $iG^{-1}_\pi(p) = p^2 - m^2 - \frac{\lambda}{6}v^2 - \frac{\lambda}{6}\int_p G_\pi(p)$

p-independent self-energy $\Rightarrow$ parametrization $G_\pi(p) = \dfrac{i}{p^2 - m_\pi^2}$

$\Longrightarrow m_\pi^2 = h/v$ Goldstone's theorem

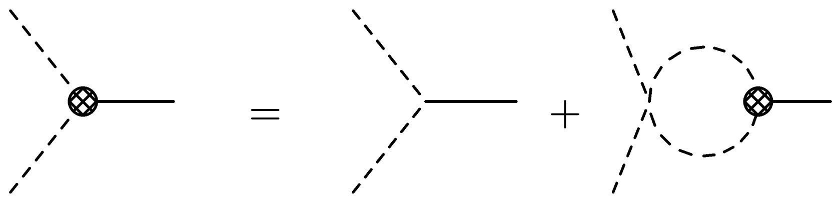
DS equation for the sigma propagator

$$iG_\sigma^{-1}(p) \quad = \quad p^2 - m_{\sigma,0}^2 - \frac{\lambda}{6}T(m_\pi) - \frac{i\lambda v}{6\sqrt{N}}\int_k G_\pi(p-q)G_\pi(k)\Gamma^{ii0}_{\pi\pi\sigma}(p-k,k,-p),$$



DS equation for the 3-point function

$$-\Gamma^{ij0}_{\pi\pi\sigma}(p,q,-p-q) \quad = \quad \frac{\lambda v}{3\sqrt{N}}\delta_{ij} + \delta_{ij}\frac{i\lambda}{6N}\int_k G_\pi(p+q-k)G_\pi(k)\Gamma^{kk0}_{\pi\pi\sigma}(p+q-k,k,-p-q),$$



iterative solution $\Gamma^{ij0}_{\pi\pi\sigma}(p,q,-p-q) = -\frac{\frac{\lambda v}{3\sqrt{N}}}{1-\frac{\lambda}{6}I(p+q)}\delta_{ij}.$

making use of the Ward identity $v\sqrt{N}\Gamma_{\pi\pi\sigma}(0,p,-p) = iG_\sigma^{-1}(p) - iG_\pi^{-1}(p)$

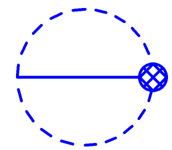
$$iG_\sigma^{-1}(p) = \underbrace{iG_\pi^{-1}(p)}_{p^2-h/\Phi} - \frac{\lambda v^2/3}{1-\frac{\lambda}{6}I(p)}$$

The $O(N)$ model at next-to-leading order

$$\frac{\delta S}{\delta \pi_i} = -\left[(\partial^2 + m^2_{\pi,0})\pi_i + \frac{\lambda}{3\sqrt{N}} v\sigma\pi_i + \frac{\lambda}{6N}\pi_i\pi_j^2 + \frac{\lambda}{6N}\pi_i\sigma^2\right]$$

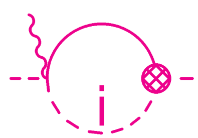
$$\frac{\delta S}{\delta \sigma} = -\left[(\partial^2 + m^2_{\sigma,0})\sigma + \frac{\lambda}{2\sqrt{N}} v\sigma^2 + \frac{\lambda}{6\sqrt{N}} v\pi_i^2 + \frac{\lambda}{6N}\sigma^3 + \frac{\lambda}{6N}\sigma\pi_i^2 + E_0(v) - \sqrt{N}h\right]$$

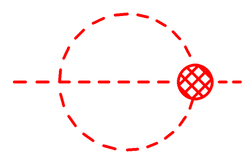
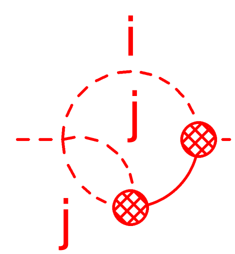
Equation of state

$$v\sqrt{N}\left[m^2 + \frac{\lambda}{6}v^2 + \frac{\lambda}{6}\left(1 - \frac{1}{N}\right)\int_p G_\pi(p) + \frac{\lambda}{2N}\int_p G_\sigma(p) + \frac{\lambda}{6vN^{3/2}}\right.$$  $$\left. - \frac{h}{v}\right] = 0$$

Pion propagator

$$-iG_\pi^{-1}(p) = -p^2 + m^2 + \frac{\lambda}{6}v^2 + \frac{\lambda}{6}\int_q G_\pi(q) + \frac{\lambda}{6N}\left[\int_q G_\sigma(q) + \int_q G_\pi(q)\right]$$

$$+ \frac{\lambda v}{3\sqrt{N}}$$ 

$$+ \frac{\lambda}{6N}$$  $$+ \frac{\lambda}{6N}$$ 

First task: combine the different diagrams to show the Goldstone's theorem

Make use of the Ward-identity $$G_\sigma(p) - G_\pi(p) = -\frac{\lambda v^2}{3} G_\sigma(p) G_\pi(p) \frac{i}{1-\frac{\lambda}{6} I(p)}$$

to obtain

$$\begin{aligned}
0 &= v\sqrt{N}\left\{ m^2 + \frac{\lambda}{6} v^2 + \frac{\lambda}{6}\int_q G_\pi^{NLO}(q) + \frac{\lambda}{6N}\left(\int_q G_\sigma(q) - \int_q G_\pi(q)\right)\right. \\
&\left. + \frac{\lambda}{3N}\int_q \frac{G_\pi(q)}{1-\frac{\lambda}{6}I(q)} + \frac{\lambda^2 v^2}{9N}\int_q \frac{-iG_\pi(q)G_\sigma(q)}{(1-\frac{\lambda}{6}I(q))^2} - \frac{h}{v}\right\} \\
-iG_\pi^{-1}(p) &= -p^2 + m^2 + \frac{\lambda}{6} v^2 + \frac{\lambda}{6}\int_q G_\pi^{NLO}(q) + \frac{\lambda}{6N}\left(\int_q G_\sigma(q) - \int_q G_\pi(q)\right) \\
&+ \frac{\lambda}{3N}\int_q \frac{G_\pi(p-q)}{1-\frac{\lambda}{6}I(q)} + \frac{\lambda^2 v^2}{9N}\int_q \frac{-iG_\pi(p-q)G_\sigma(q)}{(1-\frac{\lambda}{6}I(q))^2}
\end{aligned}$$

Goldstone's theorem fulfilled by the NLO approximation $-iG_\pi^{-1}(p=0) = \frac{h}{v}$

denominators are the result of vertex function resummation

identical equations with the ones obtained in the auxiliary field formalism using Gauss integration around the saddle point J.O. Andersen *et al.*, PRD70 (2004) 116007

Renormalization of Resummed QFT

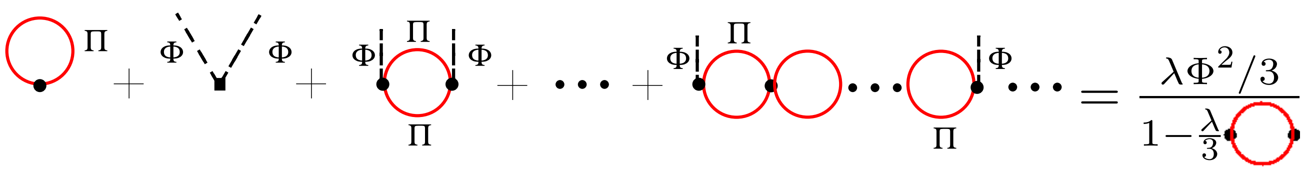
Resummation is needed

- when there is rearrangement in the ground state and spectrum
  (SSB, phase transition)

  T compensates for the power of coupling spoiling the usual loop expansion
  thermal mass $\longrightarrow$ daisy resummation

- when large N techniques are used

  infinitely many diagrams contribute at the same order of the 1/N expansion



- in non-equilibrium context to sum the secular terms

  perturbation theory works up to time $\sim 1/\lambda$

direct application of resummation methods obstructed by non-trivial relation to the order by order renormalization recently much effort was invested in the study of the renormalization

H. van Hees, J. Knoll, Phys.Rev. D65 (2002) 025010

J.-P. Blaizot, E. Iancu, U. Reinosa, Nucl. Phys. A736 (2004) 149

F. Cooper, J. F. Dawson, B. Mihaila, Phys.Rev. D70 (2004) 105008, ibid. D71 (2005) 096003

J. Berges, Sz. Borsányi, U. Reinosa, J. Serreau, Annals Phys. 320 (2005) 344

. . . . . .

key issue: resummation of counterterm diagrams

for 2PI: counterterm diagrams which remove subdivergences are generated by a Bethe–Salpeter–type equation

it is helpful having not only the equation to be renormalized but also a guiding diagrammatic expansion of it illustration $\longrightarrow$

Renormalization of $O(N)$ model at LO

EoS $$m^2 + \frac{\lambda}{6} v^2 + \frac{\lambda}{6} \int_p \frac{i}{p^2 - m_\pi^2} = \frac{h}{v} \qquad \Rightarrow \frac{h}{v} = m_\pi^2.$$

$$m^2 + \frac{\lambda}{6} v^2 + \frac{\lambda \Lambda^2}{96\pi^2} - \frac{\lambda m_\pi^2}{16\pi^2} \ln \frac{e\Lambda^2}{M_0^2} + \frac{\lambda}{6} T_F(m_\pi) = m_\pi^2 \Big| /\lambda$$

non-perturbative renormalization

$$\underbrace{\frac{m^2}{\lambda} + \frac{\Lambda^2}{96\pi^2}}_{\frac{m_R}{\lambda_R}} + \frac{v^2}{6} T_F(m_\pi) = m_\pi^2 \underbrace{\left(\frac{1}{\lambda} + \frac{1}{96\pi^2} \ln \frac{e\Lambda^2}{M_0^2} \right)}_{\frac{1}{\lambda_R}}$$

Renormalized EoS $$m_R^2 + \frac{\lambda_R}{6} v^2 + \frac{\lambda_R}{6} T_F(m_\pi) = m_\pi^2$$

behind this there is a diagrammatic expansion: supper-daisy resummation

Self-consistent equation
for the self-energy

$$\Pi(m_\pi) = \frac{\lambda}{6}\int_p \frac{i}{p^2 - m_\pi^2} = \frac{\lambda}{6}\int_p \frac{i}{p^2 - m^2 - \frac{\lambda}{6}v^2 - \Pi(m_\pi)}$$

solved iteratively

$$\Pi^{(n)} = \frac{1}{6}(\lambda + \sum_{i=1}^{n-1}\delta\lambda^{(i)})\int_p \frac{i}{p^2 - m^2 - \frac{\lambda}{6}v^2 - \Pi^{(n-1)} - \sum_{i=1}^{n-1}\delta m^{2(i)} - \frac{v^2}{6}\sum_{i=1}^{n-1}\delta\lambda^{(i)}}$$

$$+\sum_{i=1}^{n}\delta m^{2(i)} + \frac{v^2}{6}\sum_{i=1}^{n}\delta\lambda^{(i)}$$

using the expansion

$$\frac{i}{p^2 - m^2 - \frac{\lambda}{6}v^2 - \Pi} = \frac{i}{p^2 - m^2 - \frac{\lambda}{6}v^2} + \frac{i\Pi}{\left(p^2 - m^2 - \frac{\lambda}{6}v^2\right)^2} + \frac{i\Pi^2}{\left(p^2 - m^2 - \frac{\lambda}{6}v^2\right)^3} + \ldots$$

$$\lambda_{\text{bare}} = \lambda + \delta\lambda^{(1)} + \delta\lambda^{(2)} + \delta\lambda^{(3)}\ldots = \frac{\lambda}{1 - \frac{\lambda}{96\pi^2}\ln\frac{e\Lambda^2}{M_0^2}} \quad\Longrightarrow\quad \lambda^{-1} = \lambda_{\text{bare}}^{-1} + \frac{1}{96\pi^2}\ln\frac{e\Lambda^2}{M_0^2}$$

$$\delta m^{2(1)} + \delta m^{2(2)} + \delta m^{2(3)} + \ldots = -(\lambda + \delta\lambda^{(1)} + \delta\lambda^{(1)} + \ldots)\frac{\Lambda^2}{96\pi^2} + \frac{m^2}{\lambda}(\delta\lambda^{(1)} + \delta\lambda^{(2)} + \delta\lambda^{(3)}\ldots)$$

$$\frac{m^2 + \sum\delta m^2}{\lambda + \sum\delta\lambda} + \frac{\Lambda^2}{96\pi^2} = \frac{m^2}{\lambda} \quad\Longrightarrow\quad \frac{m^2_{\text{bare}}}{\lambda_{\text{bare}}} + \frac{\Lambda^2}{96\pi^2} = \frac{m^2}{\lambda}$$

Renormalization of $\Pi(m_\pi) = \frac{\lambda}{6}\int_p \frac{i}{p^2 - m_\pi^2}$

naive counterterm $\delta m^2(T) = -\frac{\lambda}{96\pi^2}\Lambda^2 + m_\pi^2 \frac{\lambda}{96\pi^2}\ln\frac{e\Lambda^2}{M_0^2}$

temperature dependent

correct counterterm at n^{th} perturbative order

$$\sum_{i=1}^{3}\delta m^{2(n)} + \frac{v^2}{6}\sum_{i=1}^{3}\delta\lambda^{(n)} = \frac{1}{6}\left(\lambda + \sum_{i=1}^{n-1}\delta\lambda^{(1)}\right)\left[-\frac{\Lambda^2}{16\pi^2} + \left(m^2 + \frac{\lambda}{6}v^2\right)\frac{1}{16\pi^2}\ln\frac{e\Lambda^2}{M_0}\right]$$

can be obtained with the replacements

$\lambda \to \lambda + \sum \delta\lambda$ and $m_\pi^2 \to m^2 + \frac{\lambda}{6}v^2$

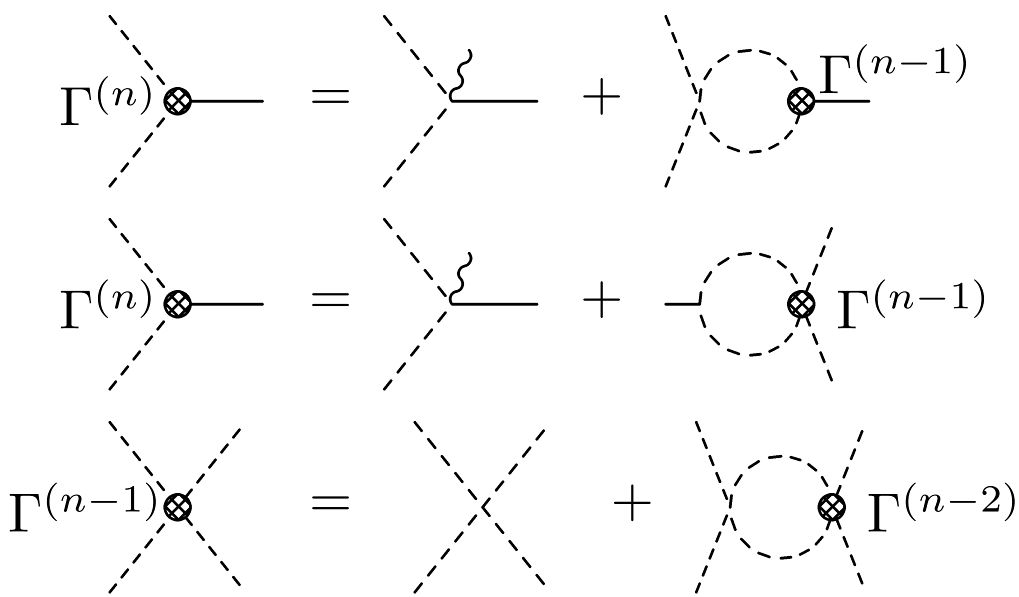
Separating the vertex function resummation

- $G_\pi(p)$ is the LO propagator $\rightarrow$ super-daisy diagrams are resummed
- at $n^{\rm th}$ iteration of the vertex the LO $G_\sigma^{(n)}(p)$ is given through the Ward identity

$$v\sqrt{N}\Gamma_{\pi\pi\sigma}(0,p,-p) = iG_\sigma^{-1}(p) - iG_\pi^{-1}(p)$$

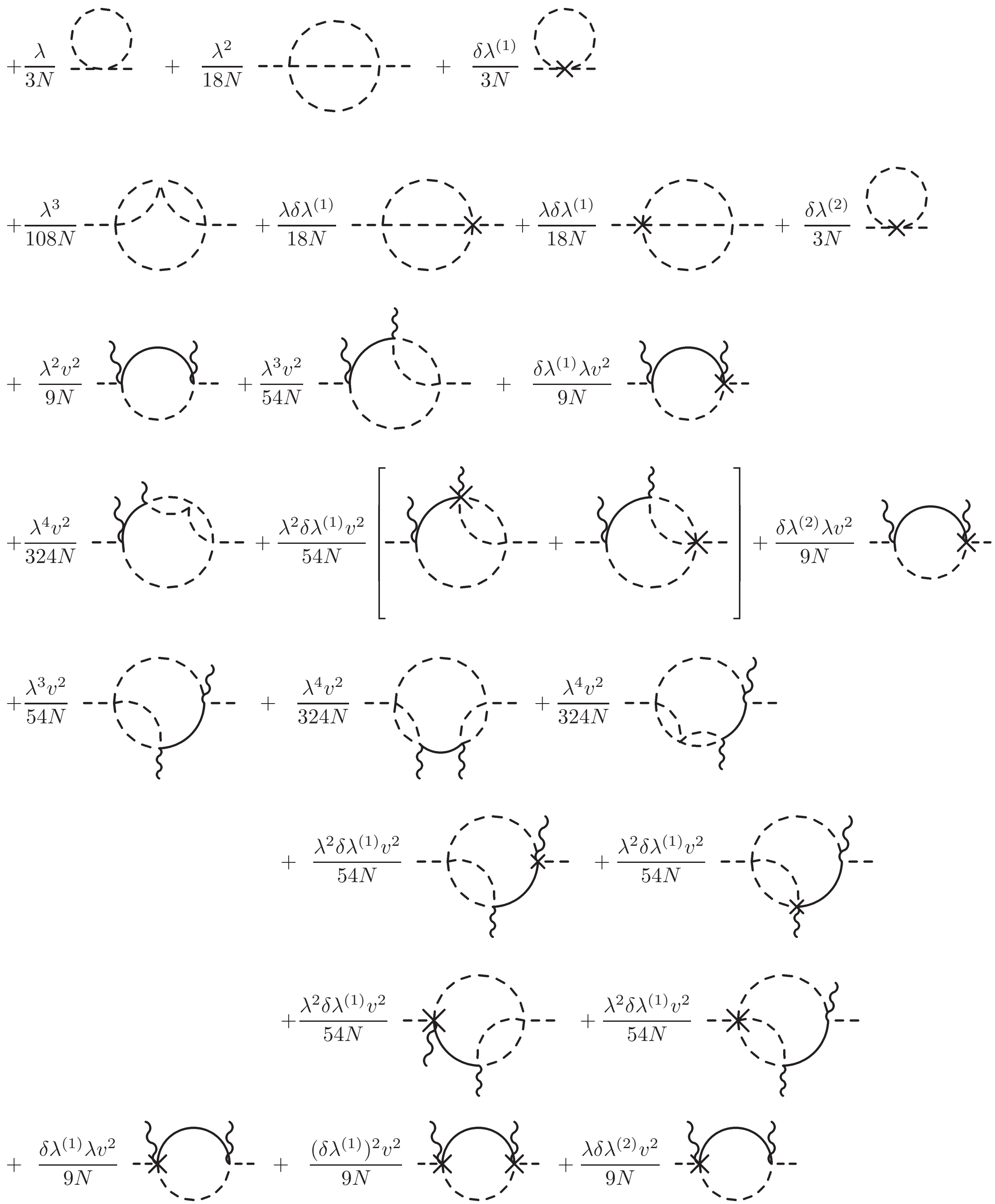
$$G_\sigma^{(n)}(p) = G_\pi(p) - \frac{i\lambda v^2}{3} G_\sigma^{(n)}(p) G_\pi(p) \left[1 + \frac{\lambda}{6} I(p) + \ldots \frac{\lambda^n}{6^n} I^n(p)\right]$$

- consistency for $\Gamma_{\pi\pi\sigma}$ requires that when its n'th iteration is used perform a $n-1$ iteration in the 4-point vertex function $\Gamma_{\pi\pi\pi\pi}$.



- because of the two-loop skeleton diagrams start with the first iteration of $\Gamma_{\pi\pi\sigma}$.

$$-iG_\pi^{-1}(p) \;=\; -p^2+m^2+\frac{\lambda}{6}v^2+\frac{\lambda}{6}\int_q G_\pi^{NLO}(q)+\frac{\lambda}{6N}\left(\int_q G_\sigma(q)-\int_q G_\pi(q)\right)$$



$T=0$ divergences of $\frac{\lambda}{3N}\int_q \frac{G_\pi(p-q)}{1-\frac{\lambda}{6}I(q)} + \frac{\lambda^2 v^2}{9N}\int_q \frac{-iG_\pi(p-q)G_\sigma(q)}{(1-\frac{\lambda}{6}I(q))^2}$

one uses that in Euclidian space

$$I_R(q) = \frac{1}{16\pi^2}\left[\ln\frac{q^2}{M_0^2} + \frac{2m^2}{q^2}\left(1+\ln\frac{q^2}{m^2}\right)\right] + \mathcal{O}(1/q^4)$$

summation of divergencies $\qquad \alpha = \frac{\lambda}{48\pi^2}$

quadratic $\qquad \frac{\lambda}{3N}\frac{M_0^2}{8\pi^2}\frac{e^{-2\alpha}}{\alpha}\mathrm{li}\left(e^{\frac{2}{\alpha}+\ln\frac{\Lambda^2}{M_0^2}}\right)$

logarithmic $\qquad -\frac{\lambda^2 v^2}{9N}\frac{1}{8\pi^2}\frac{\alpha\ln(\Lambda/M_0)}{1+\alpha\ln(\Lambda/M_0)}$

p^2-dependent $\qquad \frac{\lambda}{3N}\frac{p^2}{16\pi^2}\frac{\alpha\ln(\Lambda/M_0)}{(1+\alpha\ln(\Lambda/M_0))^2}\left[(1+\alpha)\ln\frac{\Lambda}{M_0}+1+2\alpha\right]$

m_π^2-dependent $\qquad -\frac{\lambda}{3N}\frac{m_\pi^2}{8\pi^2}\left[\frac{3}{\alpha}\ln\left(1+\alpha\ln\frac{\Lambda}{M_0}\right)+\left(1-\ln\frac{m_\pi^2}{M_0^2}\right)\frac{\alpha\ln(\Lambda/M_0)}{1+\alpha\ln(\Lambda/M_0)}\right]$

$$\int_0^{\frac{\Lambda}{M_0}} dx \frac{x^3}{\left[\left(x+\frac{b}{2}\right)^2+a^2\right]\left[\left(x-\frac{b}{2}\right)^2+a^2\right]} \frac{1}{(1+\alpha\ln x)^2}$$

$$\approx \int^{\Lambda/M_0} dx \frac{1}{x(1+\alpha\ln(x))^2} = \frac{1}{\alpha}\frac{1}{1+\alpha\ln\Lambda/M_0}$$

$$\approx \frac{\alpha\ln\Lambda/M_0}{1+\alpha\ln\Lambda/M_0} + \text{finite}$$

$$\lambda_{\text{bare}} = \lambda + \sum \delta\lambda = \lambda + \frac{\lambda^2}{96\pi^2}\frac{\ln e\Lambda^2/M_0^2}{1-\frac{\lambda}{96\pi^2}\ln e\Lambda^2/M_0^2}$$

Conclusions

- the equation of state and the pion propagator obtained at NLO of the large N approximation to Dyson-Schwinger formalism with elementary $(\sigma, \vec{\pi})$ fields
- the two equations are identical with the ones derived using auxiliary field formalism ($\sigma, \vec{\pi}$ and composite α fields)
- separation of propagator and vertex resummation done using Ward identities
- with the iteration of the vertex function the counterterm diagrams are explicitly constructed
- in contrast to the renormalization in the auxiliary field formalism, where there was no such diagrammatic expansion the renormalization procedure is more natural not leading to puzzling results

  – not only at the minimum of the potential
  – no T-dependent divergences
  – no bare couplings in the finite equations

- concrete finite temperature calculation of the diagrams and their explicit resummation still to be done